EXERCISE 1:

(a) \( \text{Var}(0.5A + 0.5B) = 0.0525 \)
\[ 0.5^2 \cdot (0.16) + 0.5^2 \cdot (0.25) + 2 \cdot 0.5 \cdot 0.5 \cdot \sigma_{AB} = 0.0525 \]
\[ \Rightarrow \sigma_{AB} = \]

(b) \( \text{Cov} \left( \sum a_i \cdot R_i , \sum b_j \cdot R_j \right) = \sum \sum a_i \cdot b_j \cdot \text{Cov} (R_i, R_j) \)

OR \( \text{Cov} (a^{' \cdot R} , b^{' \cdot R}) = E(a^{' \cdot R} - a^{' \cdot E})(b^{' \cdot R} - b^{' \cdot E}) \)
\[ = a^{' \cdot E}(R-E) (R-E) b^{' \cdot E} = a^{' \cdot E} b^{' \cdot E} \]

EXERCISE 2:

1. PORTFOLIO 2 CANNOT BE ON THE EFFICIENT FRONTIER

2. \( \bar{R}_p = \sum \bar{R}_i = \frac{1}{2} \cdot \frac{25}{2} \bar{R}_i = 0.15 \)

\[ \sigma_p^2 = \sum \sum \bar{R}_i \bar{R}_j \sigma_{ij} = \]
\[ = \frac{1}{25} \cdot (0.60^2 + 25 \cdot (24) \cdot \frac{1}{25} \cdot \frac{1}{25} \cdot (0.60)(0.60) = 0.1872 \]

AND \( \sigma_p = 0.4327 \)

IN GENERAL: \( \sigma_p^2 = \frac{s^2}{n} + \frac{n-1}{n} \cdot \sigma^2 \)

FOR EQUALLY WEIGHTED PORTFOLIO WITH ALL VARIANCES AND COVARIANCES EQUAL.
3. We want \( \sigma_p^2 = \sigma_n^2 + \frac{n-1}{n} \rho^2 \leq 0.49 \).

Solve for \( n \) to get \( n = 9 \).

4. Yes. As \( n \) gets larger \( \sigma_p^2 \rightarrow \rho^2 \).

\[ \sigma_p^2 \leq \sigma_n^2 \]

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**Exercise 3**

From class notes:

\[ x = \frac{\sum_i^1}{\sum_i^1} \]

\[ x_k = \frac{\sum V_{h_i}}{\sum \sum V_{h_i}} \]

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**Exercise 4**

\[ \sigma_p^2 = \frac{1}{n} \sigma_n^2 + \frac{n-1}{n} \rho^2 \]

It is given that \( \sigma_n^2 = 50 \) and \( \bar{\sigma}_n = 10 \).

Compute \( \sigma_p^2 \) when \( n = 5, 10, 20, 50, 100 \).

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**Exercise 5**

Min \( \alpha = \sum \xi x - 2 \bar{x} (\sum \xi x - \bar{x}) \)

\[ \frac{\partial \alpha}{\partial \bar{x}} = 2 \sum \xi x - 2 \bar{x} = 0 \Rightarrow \bar{x} = \frac{\sum \xi x}{\bar{\xi}} \]

Then,

\[ \bar{x} = \frac{1}{\bar{\xi}^2} \]

Finally,

\[ \bar{x} = \frac{\sum \xi x}{\bar{\xi}^2} \]