Exercise 1:
The call option will be exercised if $S_1 > 50$. The seller of the call will make profit if $E - S_1 + C > 0$, or $50 - S_1 + 4 > 0 \Rightarrow S_1 < 54$.

Exercise 2:
The put option will be exercised if $S_1 < 40$. The holder of the put will make profit if $E - S_1 - P > 0$ or $40 - S_1 - 3 > 0 \Rightarrow S_1 < 37$.

Exercise 3:
Let $S_1$ be the stock price at expiration.

a. Then the 2 puts will be exercised if $S_1 < 50$. Therefore for the 2 puts the profit is: $2(50 - S_1) - 12 = 88 - 2S_1$. If $S_1 \geq 50$ then the profit is -12.

b. The call will be exercised if $S_1 > 50$. Therefore for the call the profit is: $(S_1 - 50) - 5 = S_1 - 55$. If $S_1 \leq 50$ then the profit is -5.

c. The 2 puts will be exercised if $S_1 < 50$, while the call will be exercised if $S_1 > 50$. Therefore for the 2 puts the profit is $2(50 - S_1) - 17 = 83 - 2S_1$. For the call the profit is: $(S_1 - 50) - 17 = S_1 - 67$. 
Exercise 4:
Profit from writing the two calls: If $S_1 \leq 45$ the profit is 10. If $S_1 > 50$ the profit is $10 - 2(S_1 - 45) = 100 - 2S_1$.
Profit from buying one call: If $S_1 \leq 40$ the profit is -8. If $S_1 > 40$ the profit is $S_1 - 40 - 8 = S_1 - 48$.

Exercise 5:
The table that shows the payoffs for each position:

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Payoff from long call</th>
<th>Payoff from short call</th>
<th>Payoff from long put</th>
<th>Payoff from short put</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T &gt; E_2$</td>
<td>$S_T - E_1$</td>
<td>$E_2 - S_T$</td>
<td>0</td>
<td>0</td>
<td>$E_2 - E_1$</td>
</tr>
<tr>
<td>$E_1 &lt; S_T &lt; E_2$</td>
<td>$S_T - E_1$</td>
<td>0</td>
<td>$E_2 - S_T$</td>
<td>0</td>
<td>$E_2 - E_1$</td>
</tr>
<tr>
<td>$S_T &lt; E_1$</td>
<td>0</td>
<td>0</td>
<td>$E_2 - S_T$</td>
<td>$S_T - E_1$</td>
<td>$E_2 - E_1$</td>
</tr>
</tbody>
</table>

Exercise 6:
You are given: $S_0 = $50, $E = $60, $u = 1.2$, $d = \frac{1}{u} = 0.83333$, $r = 0.10$.

a. $u^k d^{10-k} > 60 \Rightarrow k = 6$.

b. At the end of the 10th period there are 11 terminal nodes on the binomial tree. The price of the stock at each terminal node is computed using $u^j d^{10-j} S_0$, $j = 0, \ldots, 10$ and the value of the call is equal to $\max[u^j d^{10-j} S_0, 0]$. 
c. See table above.
d. Price of the call at \( t = 0 \):

1. Binomial formula: We need \( p \) and \( p' \). \( p = \frac{u - d}{u + d} = \frac{1.2 - 0.83333}{1.2 + 0.83333} = 0.72727 \), and \( p' = \frac{d}{1 + r} = \frac{0.72727 \times 1.2}{1 + 0.10} = 0.79339 \).

\[
C = 50 \sum_{j=0}^{10} \binom{10}{j} p^j (1 - p)^{10-j} - \frac{60}{(1 + 0.10)^{10}} \sum_{j=0}^{10} \binom{10}{j} p^j (1 - p)^{10-j} = 27.486.
\]

2. Discounting the expected value of the call at the end of the 10th period:

\[
C = p^{10}249.587 + \binom{10}{9} p^9 (1 - p) \binom{10}{9} + \binom{10}{8} p^8 (1 - p) \binom{10}{8} + \binom{10}{7} p^7 (1 - p) \binom{10}{7} + \binom{10}{6} p^6 (1 - p) \binom{10}{6} + \binom{10}{5} p^5 (1 - p) \binom{10}{5} + \binom{10}{4} p^4 (1 - p) \binom{10}{4} + \binom{10}{3} p^3 (1 - p) \binom{10}{3} + \binom{10}{2} p^2 (1 - p) \binom{10}{2} + \binom{10}{1} p (1 - p) \binom{10}{1} + (1 - p)^{10} \]

Exercise 7:
It is given: \( u = 1.06 \) and \( d = 0.95 \). Also the risk-free interest rate is \( r = 0.05 \) with continuous compounding. We need to find \( p \)

\[
p = \frac{e^{rt} - d}{u - d} = \frac{e^{0.05 \times 10} - 0.95}{1.06 - 0.95} = 0.5680, \text{ and therefore } 1 - p = 0.4311.
\]

The price of the stock at the three terminal nodes are 56.18, 50.35, 45.125. Therefore the intrinsic value of the call will be for the top terminal node \( c_{u2} = 56.18 - 51 = 5.18 \). For the other two terminal nodes the intrinsic value of the call is 0. The value of the option at \( t = 0 \) is calculated as follows:

\[
c = \frac{5.18 \times 0.5689^2}{e^{0.05 \times 10}} = 1.635.
\]

Exercise 8:
The data are the same as exercise 2. The intrinsic value of the put at the three terminal nodes are 0, 51 - 50.35 = 0.65, and 51 - 45.125 = 5.875. Therefore the price of the put at \( t = 0 \) is calculated as follows:

\[
p = \frac{0.65(2)(0.5689)(0.4311) + 5.875(0.4311)^2}{e^{0.05 \times 10}} = 1.376
\]

The put-call parity states that:

\[
c + \frac{E}{e^{rt}} = p + s_0 \Rightarrow p = c + \frac{E}{e^{rt}} - s_0.
\]

For our problem we have:

\[
p = 1.635 + \frac{51}{e^{0.05 \times 10}} - 50 = 1.376,
\]

which verifies that the put-call parity holds.

Exercise 9:
To answer this question we need to calculate the value of the put at each node (present value of the future payoff) and compare it to the payoff from early exercise. The put value at any node will be the greater between these two values. From the diagram below we observe that at node \( C \) the payoff from immediate exercise is 51 - 47.5 = 3.5 which is larger than the value of the put (present value of payoff is 2.865). Therefore, if the put were American it would be optimal to exercise at node \( C \).
The value of the put at node $B$ is: 
\[ \frac{0.65 \times 0.4311}{e^{0.05 \frac{T}{2}}} = 0.2767. \]

The value of the put at node $C$ is 3.5.

Finally the value of the put at node $A$ is: 
\[ \frac{0.2767 \times 0.5689 + 3.5 \times 0.4311}{e^{0.05 \frac{T}{4}}} = 1.646. \]
Exercise 10:

\[ P + S_0 = C + \frac{E}{1 + r} \]

\[ S + \text{loan} = 17 + \frac{505}{1 + 0.05} \]

\[ 115 = 117 \]

Strategy: Sell Call 417

(Borrow) to buy K = 115

\[ \text{need to return } 122.9 \]

Case 1:

\[ S_t > 105 \text{ Sell at } 105 \text{ in both cases} \]

\[ P + S_0 = C + \frac{E}{1 + r} \]

At the moment → \( E = \$ \)

\[ P + E = C + \frac{E}{1 + r} \]

\[ C > P \]