Answer the following questions:

a. The weights of the minimum risk portfolio when \( n \) stocks are involved is given by \( x = \frac{\Sigma^{-1}1}{1'\Sigma^{-1}1} \). Using this formula, show that when \( n = 2 \) the weights are given by

\[
x_A = \frac{\text{Var}(R_B) - \text{Cov}(R_A, R_B)}{\text{Var}(R_A) + \text{Var}(R_B) - 2\text{Cov}(R_A, R_B)} \quad \text{and} \quad x_B = \frac{\text{Var}(R_A) - \text{Cov}(R_A, R_B)}{\text{Var}(R_A) + \text{Var}(R_B) - 2\text{Cov}(R_A, R_B)}.
\]

b. Suppose short sales are allowed and three stocks \( X, Y, Z \) are used to construct the efficient frontier. Let \( A \) and \( B \) be two portfolios on the efficient frontier with \( \bar{R}_A = 0.006, \sigma_A = 0.1, \bar{R}_B = 0.01, \sigma_B = 0.2 \) and \( \sigma_{AB} = 0.02 \). The composition of portfolio \( A \) is \( 0.53X, -0.50Y, 0.97Z \). The composition of portfolio \( B \) is \( 0.53X, -1.80Y, 2.27Z \). Find the composition of the minimum risk portfolio.

c. What is systematic and unsystematic risk? How do they affect the risk of a portfolio?

d. You are given the following:

<table>
<thead>
<tr>
<th>Stock</th>
<th>( R )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Stock B</td>
<td>0.15</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The correlation coefficient is \( \rho_{AB} \). What value of \( \rho_{AB} \) would result in an optimum portfolio of \( \frac{1}{2}A \) and \( \frac{3}{2}B \)? Assume short sales are allowed and that \( R_f = 0.05 \).

e. Assume a portfolio with three stocks \( A, B, C \). Write the expression in matrix form that computes the covariance between an equally allocated portfolio and a portfolio consisting of \((20\%, 35\%, 45\%)\) No calculations, just the expression!

f. Write an expression for the covariance between two portfolios when the single index model holds.

g. Provide all the details for the different methods for tracing out the efficient frontier when \( n \) risky stocks are involved.

h. Consider the single index model and assume you have a portfolio that consists of two stocks (1 and 2). Find an expression of the covariance between the return of the portfolio and the return of stock 1, \( \text{cov}(R_p, R_1) \).

i. Assume \( R_f = 0.05 \) and two stocks \( A, B \) with \( \bar{R}_A = 0.14, \bar{R}_B = 0.10 \). Suppose the point of tangency to the efficient frontier (the one constructed using the two stocks), consists of \( 60\%A \) and \( 40\%B \). Let’s say that you want to build a portfolio by combining the risk free asset and portfolio \( G \) to obtain expected return 11%. Determine the percentages of your investment in the risk free asset and in portfolio \( G \).

j. Use a numerical example of three stocks with a value of \( R_f \) of your choice to find the point of tangency \( G \) and then (1) combine \( G \) with \( R_f \) to find portfolio \( A \) on CAL and (2) verify that \( A \) can be obtained by using the formula for the weights \( X \) when the investor requires \( \sum_{i=1}^{n}(\bar{R}_i - R_f)x_i + R_f = E \), where \( E \) is the expected value of portfolio \( A \).