Answer the following questions:

1. Suppose that the variable $X$ follows the generalized Wiener process with drift rate $\mu_X$ and variance $\sigma^2_X$, and the variable $Y$ follows the generalized Wiener process with drift rate $\mu_Y$ and variance $\sigma^2_Y$. Initially the variable $X$ has the value $\alpha$ and the variable $Y$ the value $\beta$. What is the distribution of $X + Y$ after time $\Delta t$ if:
   a. The changes in $X$ and $Y$ in any short time interval $\Delta t$ are uncorrelated?
   b. There is a correlation $\rho$ between the changes in $X$ and $Y$ in any short time interval $\Delta t$?

2. Assume log normal property of stock prices. Estimate the annual volatility of a stock of your choice. Use daily data for a 60-day period in 2021. Please provide the exact time interval, the name of the stock, and its ticker. Assume there are 252 trading days in a calendar year.

3. Assume the following inputs: $S_0 = 48$, $E = 50$, $R_f = 0.05$ continuous interest per year, $\sigma = 0.30$ and 73 days to expiration.
   a. Use the binomial option pricing model to find the price of this European call option assuming a 30-step binomial tree.
   b. Use the Black-Scholes-Merton option pricing model to find the price of this European call option.

4. Use Ito’s lemma to find the process followed by $y = \frac{e^{r(T-t)} - s}{s}$. ($T$ is fixed.)

5. The process followed by a stock price is $dS = \mu S dt + \sigma S dz$. What is the coefficient of $dt$ followed by $Q = S^3$? What is the coefficient of $dz$ followed by $Q = S^3$. Express both answers in terms of $Q$.

6. Suppose that a financial institution offers a security that will pay off a dollar amount equal to $S^2_T$ at time $T$. Assume this security follows the lognormal distribution. What is the expected value of the payoff?

7. Provide a question that will result in the answers below:
   1. It is equal to $\text{MAX}(S_T - E, 0)$.
   2. It is equal to $-\text{MAX}(E - S_T, 0)$.
   3. It is equal to $-\text{MAX}(S_T - E, 0)$.
   4. It is equal to $\text{MAX}(E - S_T, 0)$.
   5. It is the relationship between the price of a European call and a European put written on the same underlying stock with the same expiration time and same exercise price.

8. Assume that the price $S$ of stock $A$ follows the lognormal distribution. Its current value is $50$, with expected return and volatility 12% and 30% respectively per year. What is the probability that the stock price will be larger than $80$ in two years?

9. Refer to question (8). A European put is written on stock $A$ with expiration date 6 months from now and with exercise price $60$. What is the probability that the put option will not be exercised?

10. Consider the binomial option pricing model for a European put, with exercise price $52$, current stock price $50$, $u = 1.2$, $d = 0.8$ for a 30-period binomial tree. Find the maximum number of up movements so that the put will be in the money at expiration.