by Fischer Black

Fact and Fantasy In the Use of Options

Options trading is where the action is in the securities markets these days.[1] There are some good reasons for the growing popularity of options trading, such as the fact that the brokerage charge for taking a position in options can sometimes be lower than the charge for taking an equivalent position directly in the underlying stock. But for every fact about options, there is a fantasy—a reason given for trading or not trading in options that doesn't make sense when examined carefully. It is sometimes said, for example, that covered option writers almost always gain more than they lose by writing options. This statement focuses on the premium income, and downplays the possible loss of appreciation on the stock if the option is exercised. In fact, careful study shows that an investor who writes call options against his stock will often end up with a worse position than the one he started with.

This article aims at separating fact and fantasy. It will make heavy use of the option formula developed by Black and Scholes, and of the kind of analysis that led to the formula.[2] It will make use of several refinements and extensions of the formula, some of which have been published, and some of which have not yet appeared in final form.[3]

The Option Formula

The option formula (shown in Appendix A) gives the value of a call option for any stock price and time to maturity. The simplest version of the formula assumes that the short-term interest rate and the volatility of the stock never change, and that the stock pays no dividends. Thus there are five numbers we need to calculate an option value: (1) the stock price, (2) the time to maturity, (3) the exercise price, (4) the interest rate, and (5) the volatility of the stock.

The tables in Appendix B show the values of an option with a \$40 exercise price according to the formula. The option values are arranged in three different ways, so it will be easy to see the effects of changes in any one of the four inputs other than the exercise price. The values for an option with an exercise price other than \$40 can be obtained by making proportional changes in the stock price, the exercise price, and the option value.

The big unknown in the option formula is the volatility of the underlying stock. The time to maturity and the exercise price are known. The stock price and the interest rate can be observed. But the volatility of the stock must be estimated. The past volatility of the stock is very helpful in estimating its future volatility, but is not an infallible guide. The volatility of a stock can change over time, and factors other than past volatility can be helpful in predicting it.

Note that the value of an option for a given stock price does not depend on what the stock is expected to do. An option on a stock that is expected to go up has the same value, in terms of the stock, as an option on a stock that is expected to go down. An investor who thinks the stock will go up will think both the stock and the option are underpriced. An investor who thinks the stock will go down won't buy either the stock or the option.

If the price of an option on an exchange is higher than its value, and if the formula is giving the correct value, then an investor who holds the

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1. Footnotes appear on page 72.

option should sell it, and an investor who doesn't hold the option should write it. If the price of an option on an exchange is lower than its value, and if the formula is giving the correct value, then an investor who doesn't hold the option should buy it, and an investor who has written the option should buy it back.

In other words, the rules for an option buyer are the same as the rules for an option writer. If the option is underpriced, buy it. If the option is overpriced, sell it. An option writer will generally lose if he writes underpriced options, whether he holds the underlying stock or not. His premium income on an underpriced option will be more than offset by his expected losses due to a possible increase in the price of the option over its life.

The writer's gains are the buyer's losses, and the writer's losses are the buyer's gains. If an option is overpriced when it is written, the writer is likely to gain and the buyer is likely to lose. If it is underpriced when it is written, the writer is likely to lose and the buyer is likely to gain. This is true of the writer's options position whether or not he owns the underlying stock.

The hedge ratio is the ratio of the change in the option value to the change in the stock price, for very small changes in the stock price. In Appendix B, the hedge ratios appear in parentheses, under the option values. To see the effects of larger changes in the stock price, we compare two different entries in the same table. We look down a single column at the values of the option for different stock prices.

The hedge ratio tells how to set up a "neutral hedge" between the option and the stock. A neutral hedge is one that is low in risk for small moves in the stock. The hedge ratio is the ratio of stock to option needed for a neutral hedge. For example, suppose the hedge ratio is 0.50. That means that the option value goes up or down about $0.50 when the stock goes up or down $1.00. Then a position that is short two options and long one stock or long two options and short one stock will change in value very little when the stock goes up or down $1.00.[4] The gains or losses on the long position will be offset by losses or gains on the short position. Because large moves in the stock tend to alter the hedge ratio, however, they will bring gains or losses on a fixed hedged position.

In a neutral hedge, the value of the long position need not equal the value of the short position. Suppose, in the above example, that the option price is $5 and the stock price is $40. Then the value of two options is $10, while the value of one stock is $40. The values of the two sides of the position are not even close.

Option versus Stock

The value of an option is closely related to the price of its underlying stock. For small moves in the stock over a short period of time, there is a position in the option that will give almost the same action (in dollars) as a position in the stock. The hedge ratio tells what that position is. If the hedge ratio is 0.33 at a given point in time, then three options will give the same action (in dollars) as one share of stock. This is true for both buyer and seller. Buying three option contracts gives the same action (for small price moves in the short run) as buying one round lot of stock; and selling three option contracts gives the same action as shorting one round lot of stock.

So an investor who wants the action on a stock has two ways of getting it. He can deal directly in the stock, or he can deal in the option. For equivalent dollar action, he usually has to take a larger share position in the option than he would take in the stock.

Changes in the stock price and in time to maturity cause changes in the option position that is equivalent to a stock position. As the stock price goes up, the number of option contracts needed for a position equivalent to one round lot of stock goes down. When the position is way out of the money, it may take ten contracts to give a position equivalent to one round lot. When the option is way in the money, it may take only one contract to give a position equivalent to one round lot.

Sometimes it's better for an investor who wants a position in the stock to take it directly, and sometimes it's better for him to take it via the option. If the option is underpriced—i.e., he can buy it for less than the formula says it's worth—then it may be better to buy the option than to buy the stock. If the commission on an equivalent option position is less than the commission on the stock, it may be better to deal in the option. If the investor wants a short position, it is often better to sell naked calls than to short the stock, because he may get interest on the proceeds of the sale of options. The investor may have to put up less capital to take an equivalent option position, and this can be important if he has limited capital. And finally, there may be tax reasons for dealing in the option instead of the stock.

However, these factors do not always favor options. If it takes ten $100 option contracts to get the equivalent of one round lot of stock, the commission on the ten contracts is $85, under the current commission schedule. But the commission on one round lot of a $40 stock would be about $60. So the fact that multiple option contracts are often needed to get the equivalent of a given stock position can make the option commissions higher than stock commissions.

If options are priced according to the formula, then the "net money" in a stock position is always more than the net money in an equivalent option position.[5] This means that the alternative to a long stock position is a mixture of long option positions and short-term money market instruments. For example, instead of putting up $4,000 for 100 shares of a $40 stock, the investor might pay $1,000 for two option contracts, and put the remaining $3,000 in certificates of deposit.

In the short run, the option position may go up or down just about the same amount as the stock position. In the long run, if the option position is not changed, it will do better than the stock position for large moves in the stock price, and worse than the stock position if the stock price remains about the same. If the stock ends up at the exercise price, the option will expire worthless, and the option position will be down $1,000. But if the stock goes way up, the option position will start moving up twice as fast as the stock position, and will end up with a higher value than the stock position. And if the stock goes way down, the option position will go down a maximum of $1,000, while the stock position can go down any amount up to $4,000.[6]

Thus an appropriate mixture of a long position in options with short-term money market instruments is less speculative from almost any point of view than an investment in the underlying stock. In the short run, the risk in the two positions is the same. And in the long run, the option position comes out ahead for large moves in the stock in either direction. The right mixture of options and CD's has the same expected return as the corresponding stock position, but is surely a more conservative strategy.[7]

Writing Options

Writing naked call options compared with shorting the stock is like buying call options compared with buying the stock.[8] If options are priced according to the formula, and if the hedge ratio is used to set up an equivalent position, then a short position in options will give the same gains or losses in the short run as the equivalent short position in stock. In the long run, the short option position will do better than the short stock position if the stock doesn't move much. But the short option position will do worse than the equivalent short stock position if the stock goes way up or way down and the option position is left unchanged. If buying options is less speculative than buying an equivalent amount of stock, then writing naked options is more speculative than shorting an equivalent amount of stock.

Writing options against a stock position gives the equivalent of a long position in the stock. An investor who is long 100 shares of stock and short one option contract has a position that will go up in value if the stock goes up, and down in value if the stock goes down. This position can be compared with the two alternatives we have already discussed: a position in the underlying stock without option writing, and a long position in the option.

A position with one round lot of stock long and one option contract short is less risky in the short run than a position with just the one round lot of stock. If a long position in two option contracts is equivalent in the short run to a long position in 100 shares of stock, then a position with 100 shares of stock long and one option contract short will be equivalent to a long position in only 50 shares of stock. Thus the following positions are equivalent in the short run, if the hedge ratio is 0.50: (1) a position that is long 100 shares of stock

Drawing by B. Tobey; © 1958 The New Yorker Magazine, Inc.

"If you want to make a fast buck, just go in and tell them you want to make a fast buck."

and short one option contract; (2) a position that is long 50 shares of stock; and (3) a position that is long one option contract.

These three positions involve different amounts of net money. Assuming that the stock price is $40 and the option price is $5, the net money involved in each position is: (1) $3,500; (2) $2,000; and (3) $500. To see what happens to the equivalent positions in the long run, we can assume that the investor puts up $3,500, and invests the difference between $3,500 and the net money (if any) in CD's. Thus the investment in CD's in each case is: (1) zero; (2) $1500; and (3) $3000.

The investment in CD's is the limit on the amount the investor can lose. If the stock goes to zero, the investor who buys stock and writes options will lose everything; the investor who buys stock and CD's will end up with $1,500 plus interest; and the investor who buys options and CD's will end up with $3,000 plus interest. Of these three positions, the first is the most speculative (writing covered options) and the last is the least speculative (buying options and CD's).

Writing options against a stock position is speculative because this strategy does worse for large moves in the stock than alternative strategies that have equivalent exposure to the action in the stock for small moves in the stock. If an investor writes options against his stock position and the stock doesn't move, he ends up better off than if he went to an equivalent position by selling some of his stock. But if the stock has a large move, he ends up worse off than if he went to an equivalent position by selling some of his stock.

If the formula gives the correct price for an option, and if an investor writes an option against stock at that price, he is making a fair deal. His possible gains if the stock stays at about the same price are just offset by his possible losses if the stock makes a wide move (relative to selling some of his stock instead). It is not correct to say that an investor can increase his rate of return by writing call options against his stock. In fact, he reduces his "expected return," because he creates a position that is equivalent to selling some of his stock. He creates a position in which he will come out ahead only if the stock doesn't move very much. He will come out behind if the stock moves a lot.

The only way a writer can improve expected return and retain the same exposure to small stock movements is to buy more stock and write overpriced options against his total stock position. The hedge ratio tells how much more stock to buy. If the hedge ratio is 0.50, then writing options against a stock position cuts its exposure in half; so the investor should double his stock position and write overpriced calls on all of it. If the hedge ratio is 0.33, then writing options against a stock position cuts its exposure by a third; so the investor should increase his stock position by 50 per cent, and then write overpriced calls on all of it.

Writing calls makes sense when the calls are overpriced. It does not make sense when the calls are underpriced. The fact that one position is more "speculative" than another is not important in most cases. It becomes important when a large loss has more serious consequences than loss of the money. When a large loss would lead to lawsuits and other complications, it may become important to avoid the more speculative positions. Writing overpriced calls against a stock position makes just as much sense as buying underpriced calls for most investors.

Hedging and Spreading

One way to use options is to hedge options against stock. If the hedge ratio is 0.33, then a neutral hedge will be three option contracts against one round lot of stock. A neutral hedge is achieved by going either long the stock and short the options, or short the stock and long the options. A neutral hedge is neither bullish nor bearish. If the hedge is long stock and short options, it will show gains for small moves in the stock and losses for large moves in the stock in either direction. If the hedge is long options and short stock, it will show losses for small moves in the stock and gains for large moves in the stock in either direction.[9]

When the option is underpriced, then the way to hedge is to go long options and short stock. When the option is overpriced, the way to hedge is to go long stock and short options. The farther out of line the option is, the larger the range of stock prices for which the hedge will end up profitable, and the smaller the range of stock prices for which the hedge will end up unprofitable.

Another way to use options is to spread options against one another. A "money spread" involves buying an option at one striking price and selling an option at another striking price, both on the same stock and with the same maturity. A "time spread" or "calendar spread" involves buying an option at one maturity and selling one at another; both on the same stock with the same striking price. A "butterfly spread" involves buying an option in the middle (in terms of either striking price or time) and selling one on each side, or selling an option in the middle and buying one on each side. More complicated spreads are possible too.

The hedge ratios on two options tell how to create a neutral spread between them. To find the right ratio, just divide the two hedge ratios. For example, if the hedge ratio on an October option is 0.10, and the hedge ratio on the corresponding January option is 0.30, then a neutral hedge would involve three Octobers and one January.

Once the ratio of options in a spread has been figured, the spread can be analyzed more exactly. With a 3:1 spread, we multiply the value of the first option by three, and subtract the result from the value of the second option. This is the value of the spread. Then we multiply the price of the first option by three, and subtract the result from the price of the second option. This is the price of the spread. If the value is greater than the price, it makes sense to sell the first option and buy the second. If the value is less than the price, it makes sense to buy the first option and sell the second.

A spread makes sense, of course, when the long side is underpriced and the short side is overpriced, or when the long side is more underpriced than the short side, or when the long side is less overpriced than the short side. The analysis of a spread is less sensitive to the estimated volatility of the stock than the analysis of a hedge of option against stock; an increase in the volatility estimate will increase the value of all the options on a stock. If an October option seems more overpriced than the corresponding January, and we increase the volatility estimate on the stock so that both now seem underpriced, it is likely that the October will seem less underpriced than the January. The indicated spread will be the same in either case: short the October and long the January in an appropriate ratio.

A spread that is short the near month and long a more distant month is speculative, in the sense that it makes money for small moves in the stock, and loses money for large moves in the stock. Very large moves in the stock can be disastrous if the spread is not changed. This is particularly true when the spread ratio is high. A position with one January long and ten Octobers short has great exposure to potential large losses. Similarly, a spread that is long a more in-the-money option and short a more out-of-the-money option is speculative. A position with one $60 option long and ten $80 options short has great exposure to potential large losses. The greater the difference in the exercise prices, the more speculative it is. It makes sense for an investor who is working with spreads to try to have some that are speculative and some that are conservative. Then if stocks generally move way up or way down, his gains on the conservative spreads will help offset his losses on the speculative spreads; and if stocks generally don't move very much, his gains on the speculative spreads will help offset his losses on the conservative spreads.

Estimating Volatility

The volatility of a stock can be estimated by looking at the size of the typical change in the stock price from day to day. The farther back in time we look, the more data we have to look at. If the volatility of a stock did not change, we would look as far back as possible, and would give as much weight to far distant months as to near months. But the volatility does change, so more weight should be given to recent months, and less weight should be given to distant months.

This means that when a stock seems to have a sharp increase in volatility during a month, our estimate for the future will be higher than it was, but not as high as the apparent volatility in the latest month. Sharp increases and decreases in volatility are often only temporary.

In estimating the volatility of a stock, we can also use information on the price behavior of other stocks. If the latest month shows a sharp increase in volatility for stocks generally, then it makes sense to increase our volatility estimate more for any given stock than we would if it were the only stock showing a sharp increase in volatility. If the latest month shows a sharp increase in the volatility of one of two stocks in the same industry, then the estimated volatility on the other stock should be increased too.

The direction of the price movement in a stock can also be used. A stock that drops sharply in price is likely to show a higher volatility in the future (in percentage terms) than a stock that rises sharply in price.

Sometimes other kinds of information can be useful. If we know that a company with a volatile stock is merging with a large, stable company, we may want to reduce our estimate of the future volatility of its stock. If we know that a company is starting a risky new venture, we may want to increase our estimate of the future volatility of its stock.

An increase in the estimated volatility for a stock will increase the values of all the options on the stock. Thus when the options on a stock seem generally overpriced, it is possible that the "market's estimate" of the volatility of the stock is higher than the estimate used in the formula. When the options on a stock seem generally underpriced, it is possible that the market's estimate of its volatility is lower than the estimate used in the formula. (The other possibility is that the market is simply pricing the options incorrectly.) Since the market may know some things about the future volatility in the stock that we don't know, the volatility estimate implied by the general level of option prices on a stock should be given some weight in estimating the stock's future volatility.

There are times when most traded options seem underpriced, and times when most traded options seem overpriced. Again, there are two kinds of possible explanations for this. It may be that the market is expecting volatilities to be generally lower or generally higher than the estimates used in the formula, or it may be that factors unrelated to option values are affecting the option prices.

Sometimes there should be a different volatility estimate for each option maturity. If the volatility of a stock was unusually high in the latest month, we might project a gradual decline in volatility back to more normal levels. On the other hand, if the volatility of the stock has been increasing in recent months, we might project a continued increase in volatility for a time.

Interest Rates

The interest rate in the option formula is the rate on a very low risk note that matures at the time the option expires. This means that there normally will be a different interest rate for each different option maturity. The rates on CD's and prime commercial paper are given separately for different maturities, so it makes sense to use those rates as inputs to the option formula.

Holding constant all the inputs to the option formula except the interest rate, an increase in the interest rate always increases the value of an option. To get a rough idea of why this is so, note that an increase in the interest rate reduces the present value of the exercise price. Since the exercise price is a potential liability for the holder of an option, this increases the value of the option. A one percentage point change in the interest rate does not generally have much effect on the value of an option. To see the effects of a five percentage point change in the interest rate, see Table 2 in Appendix B.

An increase in the interest rate will have a larger effect on an option with long maturity than on an option with a short maturity. Thus a change in the interest rate will change the relative values of near and far options.

An increase in the interest rate has the same effect as a reduction in the exercise price of an option, when the stock pays no dividends. A one per cent fall in the price of a CD maturing at the same time as the option has the same effect as a one per cent reduction in the exercise price.

In practice, a change in the interest rate will not occur by itself. Over the same period, there may be a change in the stock price and a change in the volatility of the stock. The change in the option price will reflect all of these changes.

Dividends

An option on a stock that pays a dividend is worth less than an option on an identical stock that pays no dividend. The higher the dividend, the less the option is worth. When a stock goes ex-dividend, the stock price usually falls, reducing the likelihood that the stock will be above its exercise price at maturity, hence the value of the option.

If the option will be exercised only at maturity, we can approximate the value of the option on a dividend paying stock by subtracting the present value of the dividends likely to be paid before maturity from the stock price. We use this adjusted stock price instead of the actual stock price in the option formula. We get the present value of the dividends by discounting them at the interest rate we are assuming. For example, if a dividend is due in three months and the interest rate is 12 per cent we discount the dividend by dividing it by about 1.03. (The number will differ slightly from 1.03 because of the effects of compounding.)

Sometimes, however, it pays to exercise an option just before it goes ex-dividend. For all dividends except the last one before the option expires, it can pay only if the annual dividend divided by the exercise price of the option is greater than the interest rate.[10] This condition is rarely satisfied. But it often pays to exercise an in-the-money option just before the last ex-dividend date. The closer the last ex-dividend date is to the expiration date of the option, the more likely it is that exercise just before the ex-dividend date will make sense.

Because there is a possibility that it will pay to exercise the option just before the last ex-dividend date, we can figure an alternative value of the option by assuming that it expires just before the last ex-dividend date. If this gives a higher value of the option than the first calculation, we will use it instead. The fact that we are using a shorter time to maturity tends to reduce the value of the option, but the fact that we are not subtracting the discounted value of the last dividend tends to increase it.

The closer the last ex-dividend date is to the maturity date, the more likely it is that the effect of leaving off the last dividend will dominate the effect of reducing the time to maturity.

Thus to figure the value of an option on a dividend-paying stock, we do two calculations of the value, and use the one that gives the higher value.[11] The first calculation subtracts the present value of all the dividends from the stock price, and uses the actual maturity date for the option. The second calculation subtracts the present value of all divi-

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dends but the last, and uses a maturity date just before the last ex-dividend date.

The holder of an option should be careful to decide whether it will pay to hold it beyond the last ex-dividend date. When the stock price is well above the exercise price just before the last ex-dividend date, it will often pay either to exercise or to sell out. This means that the writer should decide whether he wants to close his position by buying in or by having the option exercised against him. If he doesn't buy in before the stock goes ex-dividend, he may find that he has been assigned an exercise notice.

The holder of an option should keep it beyond the last ex-dividend date only if it is worth more "alive" than "dead." The value of the option alive will be its value with the stock price reduced by the dividend, with a time to maturity equal to the time between the last ex-dividend date and the date the option expires. The value of the option dead will be the stock price minus the exercise price.

The writer of an option should decide whether to buy in before the last ex-dividend date on the same basis. If he wants to avoid an exercise notice, and the option will be worth more dead than alive on the ex-dividend date, he should buy in before that date—probably several days before, because the exercise notices will start coming in faster several days before the ex-dividend date.

Transaction Costs

The transaction costs for trading in options will often be lower than the costs of making an equivalent trade in the underlying stocks. For example, assume that a six-month option with a $40 exercise price sells for $5 when the stock price is $40, and has a hedge ratio of 0.60. The round trip public commission for taking a position in the stock is now three per cent or more of the value of the stock. For a round lot of stock, this comes to $120.

To get a position that is equivalent to one round lot of stock in the short run, we need less than two option contracts. (If the hedge ratio were 0.50, we would need exactly two.) The public commission on two option contracts with a premium of $500 for each contract is $37 one way. If the option expires worthless, no further commission will be paid. If it ends up in the money, there will be a commission on the closing transaction. If it is exercised, the commission is based on the exercise price, but is no more than $65.

Taking all these possibilities into account, we get a round trip commission for the option position of around $60, which is about half the commission cost of dealing in the stock directly. This saving, however, applies only to short-term trading. If we try to take the equivalent of a long-term position in the stock by buying or selling options repeatedly, we will end up paying much more in commissions than we would in the stock.

Another element of the transaction cost on either options or stock is the market maker's or specialist's spread: the difference between the highest bid price and the lowest asked price.[12] We should use the hedge ratio to compare spreads on options with spreads on stock, just as we used it to compare commissions. When the hedge ratio is 0.50 on an option, we should compare twice the spread in the option market with the spread on the underlying stock on the stock exchange. Since it takes two option contracts to give the same action as one round lot of stock in the short run, we need to compare the spread for two option contracts with the spread for one round lot of stock.

It would not be surprising to find that the market makers' and specialists' spreads are higher on the options market than the spreads for equivalent positions in the stock market, because "information trading" may tend to shift from the market for a stock to the market for its options.

One reason why a market maker's or specialist's buying price is lower than his selling price is that he doesn't know what information those who trade with him may have. Many of them will have information about the stock that he doesn't have. He does know, however, that those who want to sell to him probably have negative information, and those who want to buy from him probably have positive information. So he protects himself to some extent by quoting a lower price to those who want to sell and a higher price to those who want to buy. He will still lose money trading with those who have important pieces of information, and this will cut into the profits he makes trading with those who do not have valuable pieces of information.

Since an investor can usually get more action for a given investment in options than he can by investing directly in the underlying stock, he may choose to deal in options when he feels he has an especially important piece of information. Also, it is easier to take a short position by writing options than by shorting the underlying stock. So many information traders will go to the options market rather than to the stock market. And many potential information traders will trade on the options market when they wouldn't bother to trade at all if the options market did not exist.

This means that in some cases a market maker or specialist will face a more dangerous trading environment on an options exchange than the specialist on the same stock faces on a stock exchange. A higher proportion of those the market maker trades with will have information that can hurt him. So he may have to set a higher bid-asked spread than the specialist in the stock does (for a corresponding position) just to break even.

The fact that the options market brings out information traders who wouldn't otherwise trade means that the market for the stock will be more efficient than it would be without the options market. Even if a piece of information shows up first on the options market, hedgers will rapidly cause the information to be incorporated in stock prices. Options trading will improve the market in the underlying stock, even if it reduces the volume of trading in the underlying stock. But because hedging brings new trading in the stock, it is unlikely that options trading will reduce the volume of trading in the underlying stock.[13]

Taxes

It appears that gains and losses for an option buyer will be taxed as capital gains and losses, while gains and losses for an option writer who buys his options back will be taxed as ordinary income or loss. A buyer who exercises his option defers realizing a gain or loss, and a writer who has an option exercised against him realizes a capital gain or loss rather than ordinary income or loss.

If everyone were in a 70 per cent tax bracket, then option values would be lower than the values given by the formula. If everyone were in the same tax bracket, then the higher the common tax bracket, the lower the option values would be. The effects of taxes are similar to the effects of dividends on the underlying stock. High taxes reduce option values, and make it profitable sometimes for an option buyer to exercise his option before it expires.

To get a rough idea of the amount by which taxes can affect option values, we can calculate a discount factor as follows. We multiply the interest rate by the common tax bracket. We multiply this by the fraction of a year that remains before the option expires, and add 1.0. We divide the stock price by this discount factor before applying the option formula. Note that the discount factor is applied to the stock price, not to the option value directly.

When the interest rate is 12 per cent, the effect on option values of assuming a common tax bracket of 50 per cent is similar to the effect of assuming that the stock pays a six per cent dividend. The effect is not huge, but it is sometimes significant.

When tax brackets differ, an option will have a different value for an investor in a high tax bracket than for an investor in a low tax bracket. The value will be lower for the investor in a high tax bracket. This means that investors in high tax brackets should more often be writers of options, and investors in low tax brackets should more often be buyers of options. This is true in spite of the fact that investors in high tax brackets who buy options may get the benefit of capital gains taxation of their gains. Investors in low tax brackets also get the benefit of capital gains treatment, and they are taxed at lower rates.

In particular, it means that tax exempt institutions would generally be better off buying options than writing them, so long as they are not overpriced. Hopefully, the push to allow institutions to write options freely will be extended so that they will be allowed to buy options, too. While buying options is often considered imprudent, we have already noted that a mixture of options and CD's is actually less speculative than an equivalent combination of holding stock and writing options.

If the price of an option is \$6, it may be worth \$7 to a tax-exempt investor, and \$5 to a taxable investor. When the taxable investor writes the option at \$6, he makes \$1. When the tax-exempt investor buys the option at \$6, he makes \$1. They both gain at the expense of the government.

Taxes affect the hedge ratios, too. An investor in a high tax bracket who wants a neutral hedge that is long stock and short options will have to write more options than an investor in a low tax bracket. Suppose, for example, that the hedge ratio is 0.50. A tax exempt investor would buy one round lot of stock and write two option contracts. But an investor in a 50 per cent tax bracket might write three or four option contracts. If he is going to continue to hold the stock, then he won't realize any gains or losses from the stock. But he will realize his gains or losses from the option. If the option goes up, he will realize an ordinary loss, so the government will pay for half of the loss. If the option goes down, he will realize ordinary income, so the government will take half the gain. In effect, the government is taking half the risk. So he needs to write twice as many options to get the same after-tax risk he would have if he were tax exempt. To get a neutral hedge, he writes four option contracts instead of two against one round lot.

If an investor in a high tax bracket writes in-the-money options, he has an additional possible advantage. If the option goes up, he will have an ordinary loss. If the option expires worthless, he will have ordinary income. But if the option goes down but ends up in the money, he can let the option be exercised against him. He can either deliver the stock he holds or buy new stock in the market, and thus realize a capital gain or loss instead of ordinary income.

To see how options can be used to save taxes, let us consider an extreme example: an option with a zero exercise price. This will allow us to illustrate the principles involved in a relatively simple manner. We will assume a $40 stock, and an option that expires in three months with an exercise price of zero. We will assume that the option can be exercised at any time, so it will have a value equal to the stock price at all times. If the stock goes ex-dividend, the option will be exercised before the ex-dividend day. We will assume no dividends.

Now suppose an investor in a 50 per cent tax bracket owns 100 shares of stock and sells options on 200 shares. He is selling options on more shares than he owns. Suppose further that he plans to keep the stock, but to buy back the options just before they expire. The gains or losses on his stock will remain unrealized, while the gains or losses on the options will become ordinary income or loss. Thus his after-tax gains or losses on the options will be only half of his before-tax gains or losses. Since he has written options on two shares of stock for each share he owns, his position is perfectly hedged. Taking taxes into account, his gains or losses on the options will exactly offset his unrealized gains or losses on the stock.

But the option premiums total $8000, while the investor has only $4000 in the stock. He has $4000 to work with until the options are exercised or until he buys them back. He can invest that $4000. His gain is the interest on $4000 for the life of the option. And he gets that without bearing any risk at all. In effect, he gets an interest-free loan of $4000 for the life of the option. The equity in his hedged position is negative. Of course, he gets this benefit only if he gets interest on the proceeds of the options he wrote naked. If he doesn't get the benefit, his brokerage firm will.

But there is more. The investor doesn't have to buy back all his options at the end. If the price of the stock is lower than it was when the options were sold, he may want to let the options on 100 shares be exercised. This will give him a capital gain or loss, depending on his cost basis for the shares. If it is a capital loss, he is clearly better off letting the options on 100 shares be exercised. If it is a capital gain, he may be better off letting them be exercised, depending on the exact amount of the gain, on whether it is long term or short term, and on how long he is likely to let the gain go unrealized if he buys the options back.

These tax benefits can be shared with the buyer of the options, especially if the buyer is an investor in a low tax bracket. One way to do this is to deal in options that have an exercise price that is not zero. For example, suppose we have an option on a $40 stock with an exercise price of $10, and suppose that both writer and buyer agree that all transactions in the option will be at the stock price minus the exercise price. A tax-exempt buyer gets the action on a $40 stock for only $30. In effect, he gets a $10 interest-free loan. His gain is the interest on $10 for each share of stock.

Suppose the writer also wants the action on 100 shares. Assuming he will buy back his options at the end, he can accomplish this by buying 200 shares of stock and writing options on 200 shares. Since his gains and losses on the options are taxable at 50 per cent, the options on 200 shares only offset the gains and losses on 100 shares of stock, so he is left with the action on 100 shares. Since the options sell initially for $3000 per 100 shares, he gets $6000 for the options, and pays $8000 for the stock. His net investment is $2000 for the action on $4000 worth of stock. In effect, he is getting a $2000 interest-free loan. His gain is the interest on $2000. Since this interest is taxable, his net gain is the interest on $1000. This is the same as the tax-exempt investor's gain.

Thus both parties to the transaction get an extra three per cent after taxes on their equity, when the interest rate is 12 per cent. They get the return on the stock plus an extra three per cent. And this doesn't count the substantial extra gain that the taxable investor may get by letting his options be exercised if the stock goes down. This should be enough to show, however, that the potential tax advantages from the use of options are truly enormous.

These factors operate on listed options, too. They probably explain why options that are well in-the-money (the stock price is well above the exercise price) often sell at tangible value (stock price minus exercise price). At that price, the option is a clear bargain for a tax-exempt investor. And a taxable investor may be able to make money writing it at the same time.

For an option sold with an exercise price equal to the stock price at the time, the right to let the option be exercised is not worth much to the writer. He will gain if the stock ends up close to the initial price (but above it). If the initial price is $40, and the option sold for $5, and the stock ends up just above $40, then the writer's ordinary income if he buys the option back is just under $5. But if he lets the option be exercised, and if he has held the stock for more than six months, he will have a long-term capital gain of $5. The profit is the same, but it is taxed as long-term capital gains, rather than as ordinary income. But he gets no benefit if the stock ends up above $45 or below

$40. If it's below $40, the option won't be exercised.

But even an option sold with an exercise price equal to the stock price at the time has a tax value to a writer in a high tax bracket. In general, high bracket investors should write options, and low bracket investors should buy them.

Individual listed options may be priced at times so high that even a tax-exempt investor should write them; or so low that even a taxable investor should buy them. The option formula may help in identifying such cases. The ideal strategy might be to use the formula to help taxable investors sell overpriced options and to help tax-exempt investors buy underpriced options.

Margin Requirements

Brokers cannot lend money for the purchase of options. But this is often not a problem, because to get the equivalent of buying a stock on margin, the investor may want to hold a mixture of options and short-term money market instruments. In that case he won't want to borrow to buy options.

The investor who wants the equivalent of a short position in the stock may be much better off writing options than shorting the stock directly. If he can keep both the amount he puts up in margin and the premiums he receives in government securities, then he will be earning interest. Even if he loses interest on one or both of these amounts, they are normally smaller than the amounts he loses interest on if he shorts the stock.

Further, a taxable investor who writes naked options realizes ordinary losses if he is wrong, while an investor who shorts the stock realizes short-term capital losses if he is wrong. Ordinary losses can be used without limit, while capital losses can only be used to offset capital gains plus a small amount of ordinary income. So writing options will be better than shorting stock for both margin reasons and tax reasons.

Actual Prices

The actual prices on listed options tend to differ in certain systematic ways from the values given by the formula. Options that are way out of the money tend to be overpriced, and options that are way into the money tend to be underpriced. Options with less than three months to maturity tend to be overpriced.

Thus the money spreads that make sense usually involve buying an option with a lower exercise price and selling a corresponding option with a higher exercise price. The time spreads that make sense usually involve buying an option with a longer maturity and selling an option with a shorter maturity. The stock option hedges that seem most profitable involve buying the stock and selling out-of-the-money options with only a little time left to go. At least this has been true so far. As time goes on, the pattern may change.

One possible explanation for this pattern is that we have left someting out of the formula. Perhaps if we assumed a more complicated pattern for changes in a stock's volatility, or for changes in interest rates, we would be able to explain the overpricing of out-of-the-money options. But this seems unlikely to give a complete explanation. The underpricing of way in-the-money options is so extreme that they often sell at "parity," where the option price is approximately equal to the stock price minus the exercise price. This means that the market is not giving the remaining time to maturity any value at all. Only tax factors, as discussed above, seem to have any chance of explaining this.

Another possible explanation for the observed pattern is that market makers and other investors have to be induced to take the indicated positions by the promise of substantial profits, because they are so speculative. A market maker who buys one option contract for $1,000 and sells ten contracts at a higher exercise price for $12.50 each is taking a great risk. A sudden large move in the stock price in either direction may give him great losses. If the stock moves down, he can lose all the money he put up. If the stock moves up, he can lose more than all the money he put up.

Those who buy short, out-of-the-money options and mix them with CD's have conservative positions. At most, they can lose all the money they put into the options. Those who write short, out-of-the-money options have speculative positions, even if they hold the underlying stock. They can lose all the money they have invested, and sometimes more. So it may be that those who want conservative positions must pay others to take the speculative positions.

Still another possible explanation is that options give a form of leverage that is otherwise unavailable because of margin restrictions, and that investors bid up their prices because of the leverage they get. This explanation seems very unlikely for two reasons. First, it can't explain why in-the-money options are usually underpriced. They can offer more leverage than the investor can get directly, yet they are underpriced. And second, it can't explain why competition among writers doesn't eliminate the overpricing of out-of-the-money options. There are great numbers of investors who have as much leverage as they want, and would be glad to earn extra money by writing overpriced options if writing options were not so speculative.

At the moment, I think we have to say that we don't know why some kinds of options are consistently overpriced according to the formula and others are consistently underpriced.

Regulation

The SEC seems to be doing a good job in regulating options trading. It has allowed the creation of exchanges that are more advanced in several respects than existing stock exchanges. Clearing facilities are more modern and lower in cost, and short positions are as easy to open as long positions.

However, the Commission has imposed restrictions on the trading of options that are way out of the money. It is hard to see how this can be in the public interest.

Apparently the Commission feels that those who buy way out-of-the-money options are throwing their money away, because there is so little chance that they will be worth anything at maturity. And in fact, the formula suggests that such options are usually overpriced. But we have already noted that careful buyers of such options are actually bearing less risk of catastrophic loss than those who write the options. If the SEC restricts trading, it will hurt those who buy options to reduce their exposure to loss as well as those who buy options to increase their potential gains. Further, no one has to buy these options. Anyone who wants to can write them instead. So this seems like a situation where any possible problems can be adequately solved through additional disclosure. ■

APPENDIX A. THE FORMULA

The simple option formula is Equation 13 on page 644 of the article by Black and Scholes (1973):

$$w(x, t) = xN(d_1) - ce^{r(t-t^*)}N(d_2)$$

$$d_1 = \frac{ln\frac{x}{c} + (r + \frac{1}{2}v^2)(t^*-t)}{v\sqrt{t^*-t}}$$

$$d_2 = \frac{ln\frac{x}{c} + (r - \frac{1}{2}v^2)(t^*-t)}{v\sqrt{t^*-t}}$$

In this formula, x is the stock price, c is the exercise price, t is the current time, t^* is the time at which the option expires, r is the interest rate, v^2 is the variance rate of the return on the stock (a measure of the stock's volatility), ln is the natural logarithm, N(d) is the cumulative normal density function, and w(x, t) is the value of the option at time t when the stock price is x.

The "hedge ratio" is given by Equation 14 on page 645 of the article by Black and Scholes (1973):

$$w_1(x, t) = N(d_1)$$

In this formula, $w_1(x, t)$ is the derivative of the option formula with respect to the stock price. It is the change in the option value for a small change in the stock price, divided by the change in the stock price. Or it is the number of round lots of stock needed to balance one option contract to create a hedged position. The other symbols in this formula are defined as they are for the option formula.

APPENDIX B. TABLES OF OPTION VALUES AND HEDGE RATIOS

The tables given below show the values and hedge ratios for an option with an exercise price of $40, for different values of the other inputs to the option formula. There are six different stock prices, three maturities, three interest rates, and nine values of the volatility (annual standard deviation). The hedge ratios are in parentheses.

Table 1 makes it easy to see the influence of stock price and maturity. Reading down, we see how the value of an option changes as the stock price increases. Reading across, we see how the value changes as the maturity increases. An increase in either the stock price or the maturity will always increase the value of the option. Note that while a change of several months in maturity has a substantial effect on the value of an option, a change of a few days or a week usually has only a modest effect on the option value.

Table 2 makes it easy to see the influence of stock price and the interest rate. Reading across, we see how the value changes as the interest rate increases. An increase in the interest rate will always increase the value of the option. Note that even a five percentage point change in the interest rate usually has only a modest effect on the option value. The greatest effect of the interest rate occurs in the longest maturity options.

Table 3 makes it easy to see the influence of stock price and the volatility of the stock. Reading across, we see how the value changes as the volatility increases. An increase in the volatility will always increase the value of the option. Note that when the stock price is well below the exercise price, an increase in the volatility of the stock causes a very large percentage increase in the value of the option.

The figures in Table 2 and 3 are the same as the figures in Table 1. They are just arranged differently. →

TABLE 1. Option Values and Hedge Ratios for Different Stock Prices and Maturities, by Standard Deviation and Interest Rate.

Annual Std Dev = 0.20 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	0.00	0.01	0.07	0.00	0.02	0.12	0.00	0.04	0.20
	(0.00)	(0.01)	(0.04)	(0.00)	(0.02)	(0.06)	(0.00)	(0.03)	(0.09)
32.	0.02	0.18	0.44	0.03	0.26	0.65	0.04	0.37	0.93
	(0.02)	(0.09)	(0.16)	(0.03)	(0.12)	(0.22)	(0.04)	(0.16)	(0.29)
36.	0.36	0.94	1.50	0.45	1.22	2.01	0.55	1.55	2.61
	(0.19)	(0.31)	(0.38)	(0.23)	(0.37)	(0.46)	(0.26)	(0.44)	(0.55)
40.	1.85	2.76	3.51	2.12	3.31	4.35	2.41	3.92	5.27
	(0.57)	(0.60)	(0.62)	(0.62)	(0.66)	(0.70)	(0.66)	(0.73)	(0.77)
44.	4.80	5.63	6.38	5.22	6.41	7.49	5.65	7.21	8.64
	(0.87)	(0.82)	(0.80)	(0.89)	(0.86)	(0.86)	(0.92)	(0.90)	(0.90)
48.	8.54	9.18	9.83	9.02	10.08	11.11	9.49	10.97	12.39
	(0.98)	(0.94)	(0.91)	(0.98)	(0.96)	(0.94)	(0.99)	(0.97)	(0.96)
52.	12.50	13.04	13.60	12.99	13.98	14.96	13.47	14.91	16.30
	(1.00)	(0.98)	(0.97)	(1.00)	(0.99)	(0.98)	(1.00)	(0.99)	(0.99)

Annual Std Dev = 0.30 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	0.02	0.18	0.45	0.02	0.23	0.59	0.03	0.29	0.76
	(0.01)	(0.07)	(0.14)	(0.02)	(0.09)	(0.17)	(0.02)	(0.11)	(0.21)
32.	0.19	0.70	1.27	0.23	0.86	1.58	0.27	1.04	1.94
	(0.09)	(0.20)	(0.28)	(0.11)	(0.24)	(0.33)	(0.12)	(0.28)	(0.38)
36.	0.92	1.89	2.72	1.05	2.21	3.25	1.19	2.56	3.84
	(0.29)	(0.39)	(0.45)	(0.32)	(0.44)	(0.51)	(0.35)	(0.49)	(0.56)
40.	2.63	3.85	4.84	2.89	4.36	5.59	3.16	4.90	6.40
	(0.56)	(0.59)	(0.61)	(0.60)	(0.63)	(0.66)	(0.63)	(0.68)	(0.71)
44.	5.36	6.55	7.54	5.73	7.22	8.50	6.11	7.91	9.49
	(0.79)	(0.75)	(0.74)	(0.81)	(0.79)	(0.78)	(0.83)	(0.82)	(0.82)
48.	8.79	9.78	10.70	9.23	10.58	11.82	9.68	11.38	12.94
	(0.92)	(0.86)	(0.84)	(0.93)	(0.89)	(0.87)	(0.94)	(0.91)	(0.90)
52.	12.59	13.37	14.18	13.06	14.24	15.41	13.53	15.11	16.63
	(0.97)	(0.93)	(0.90)	(0.98)	(0.94)	(0.92)	(0.98)	(0.96)	(0.94)

Annual Std Dev = 0.40 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	0.12	0.55	1.09	0.13	0.65	1.30	0.15	0.76	1.54
	(0.05)	(0.15)	(0.23)	(0.06)	(0.17)	(0.26)	(0.07)	(0.20)	(0.30)
32.	0.53	1.42	2.26	0.60	1.62	2.62	0.67	1.85	3.02
	(0.17)	(0.29)	(0.36)	(0.19)	(0.32)	(0.40)	(0.20)	(0.35)	(0.44)
36.	1.57	2.88	3.96	1.72	3.22	4.49	1.87	3.58	5.06
	(0.36)	(0.44)	(0.49)	(0.38)	(0.48)	(0.53)	(0.41)	(0.51)	(0.58)
40.	3.42	4.95	6.17	3.67	5.43	6.87	3.92	5.93	7.60
	(0.56)	(0.59)	(0.61)	(0.59)	(0.62)	(0.65)	(0.61)	(0.66)	(0.69)
44.	6.04	7.57	8.82	6.38	8.18	9.67	6.72	8.80	10.56
	(0.74)	(0.71)	(0.71)	(0.76)	(0.74)	(0.75)	(0.78)	(0.77)	(0.78)
48.	9.26	10.63	11.83	9.66	11.34	12.82	10.06	12.06	13.82
	(0.86)	(0.81)	(0.79)	(0.87)	(0.83)	(0.82)	(0.88)	(0.85)	(0.85)
52.	12.85	14.01	15.11	13.29	14.80	16.21	13.73	15.59	17.31
	(0.93)	(0.88)	(0.85)	(0.94)	(0.89)	(0.87)	(0.95)	(0.91)	(0.90)

Annual Std Dev = 0.50 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	0.32	1.08	1.88	0.35	1.22	2.14	0.39	1.37	2.42
	(0.11)	(0.22)	(0.30)	(0.11)	(0.24)	(0.33)	(0.12)	(0.27)	(0.36)
32.	0.99	2.23	3.32	1.07	2.46	3.71	1.16	2.71	4.12
	(0.24)	(0.35)	(0.42)	(0.25)	(0.38)	(0.45)	(0.27)	(0.40)	(0.48)
36.	2.26	3.89	5.20	2.41	4.23	5.73	2.58	4.59	6.28
	(0.40)	(0.48)	(0.52)	(0.42)	(0.51)	(0.56)	(0.44)	(0.54)	(0.59)
40.	4.21	6.05	7.49	4.44	6.51	8.15	4.69	6.97	8.83
	(0.57)	(0.60)	(0.62)	(0.59)	(0.62)	(0.65)	(0.61)	(0.65)	(0.68)
44.	6.78	8.65	10.13	7.09	9.21	10.92	7.40	9.77	11.72
	(0.71)	(0.70)	(0.70)	(0.73)	(0.72)	(0.73)	(0.74)	(0.74)	(0.76)
48.	9.85	11.60	13.07	10.22	12.25	13.96	10.59	12.91	14.87
	(0.82)	(0.78)	(0.77)	(0.83)	(0.80)	(0.79)	(0.84)	(0.82)	(0.82)
52.	13.27	14.84	16.24	13.68	15.56	17.23	14.10	16.29	18.22
	(0.89)	(0.84)	(0.82)	(0.90)	(0.86)	(0.84)	(0.91)	(0.87)	(0.86)

TABLE 1. Option Values and Hedge Ratios for Different Stock Prices and Maturities, by Standard Deviation and Interest Rate. (Continued)

Annual Std Dev = 0.60 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	0.62	1.72	2.76	0.67	1.88	3.05	0.72	2.05	3.35
	(0.16)	(0.28)	(0.36)	(0.17)	(0.30)	(0.39)	(0.18)	(0.33)	(0.42)
32.	1.51	3.09	4.41	1.61	3.33	4.81	1.71	3.59	5.22
	(0.29)	(0.40)	(0.46)	(0.30)	(0.42)	(0.49)	(0.32)	(0.45)	(0.52)
36.	2.96	4.91	6.44	3.12	5.25	6.95	3.29	5.60	7.48
	(0.44)	(0.51)	(0.55)	(0.45)	(0.53)	(0.58)	(0.47)	(0.56)	(0.61)
40.	4.99	7.14	8.81	5.22	7.58	9.43	5.45	8.02	10.07
	(0.58)	(0.61)	(0.63)	(0.59)	(0.63)	(0.66)	(0.61)	(0.65)	(0.68)
44.	7.54	9.74	11.46	7.83	10.26	12.19	8.13	10.79	12.93
	(0.69)	(0.69)	(0.70)	(0.71)	(0.71)	(0.72)	(0.72)	(0.73)	(0.75)
48.	10.52	12.64	14.37	10.86	13.24	15.19	11.20	13.84	16.02
	(0.79)	(0.76)	(0.75)	(0.80)	(0.78)	(0.77)	(0.81)	(0.79)	(0.80)
52.	13.81	15.79	17.47	14.20	16.45	18.38	14.58	17.12	19.28
	(0.86)	(0.81)	(0.80)	(0.87)	(0.83)	(0.82)	(0.87)	(0.84)	(0.84)

Annual Std Dev = 0.70 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	0.99	2.42	3.69	1.05	2.60	3.99	1.12	2.78	4.31
	(0.21)	(0.34)	(0.41)	(0.22)	(0.35)	(0.44)	(0.23)	(0.37)	(0.46)
32.	2.08	3.97	5.51	2.19	4.23	5.91	2.30	4.49	6.33
	(0.33)	(0.44)	(0.50)	(0.35)	(0.46)	(0.52)	(0.36)	(0.48)	(0.55)
36.	3.67	5.92	6.67	3.84	6.26	8.16	4.01	6.60	8.67
	(0.46)	(0.53)	(0.58)	(0.48)	(0.55)	(0.60)	(0.49)	(0.57)	(0.62)
40.	5.77	8.23	10.11	6.00	8.64	10.70	6.22	9.07	11.30
	(0.58)	(0.62)	(0.64)	(0.60)	(0.64)	(0.67)	(0.61)	(0.66)	(0.69)
44.	8.32	10.84	12.79	8.59	11.33	13.47	8.87	11.83	14.16
	(0.69)	(0.69)	(0.70)	(0.70)	(0.71)	(0.72)	(0.71)	(0.72)	(0.74)
48.	11.23	13.72	15.69	11.55	14.27	16.45	11.88	14.83	17.21
	(0.77)	(0.75)	(0.75)	(0.78)	(0.76)	(0.77)	(0.79)	(0.78)	(0.79)
52.	14.44	16.81	18.76	14.80	17.42	19.59	15.16	18.04	20.43
	(0.83)	(0.80)	(0.79)	(0.84)	(0.81)	(0.80)	(0.85)	(0.82)	(0.82)

Annual Std Dev = 0.80 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	1.42	3.16	4.64	1.49	3.35	4.95	1.57	3.55	5.27
	(0.25)	(0.38)	(0.45)	(0.26)	(0.40)	(0.48)	(0.27)	(0.41)	(0.50)
32.	2.67	4.87	6.62	2.79	5.13	7.01	2.91	5.40	7.42
	(0.37)	(0.47)	(0.53)	(0.38)	(0.49)	(0.55)	(0.40)	(0.51)	(0.57)
36.	4.39	6.93	8.88	4.56	7.26	9.36	4.73	7.60	9.85
	(0.49)	(0.56)	(0.60)	(0.50)	(0.57)	(0.62)	(0.51)	(0.59)	(0.64)
40.	6.55	9.30	11.39	6.77	9.70	11.95	6.99	10.10	12.52
	(0.59)	(0.63)	(0.66)	(0.60)	(0.64)	(0.68)	(0.62)	(0.66)	(0.69)
44.	9.10	11.94	14.11	9.36	12.40	14.75	9.63	12.87	15.39
	(0.68)	(0.69)	(0.70)	(0.69)	(0.71)	(0.72)	(0.70)	(0.72)	(0.74)
48.	11.98	14.81	17.02	12.28	15.33	17.73	12.59	15.85	18.44
	(0.75)	(0.74)	(0.75)	(0.76)	(0.76)	(0.76)	(0.77)	(0.77)	(0.78)
52.	15.11	17.86	20.08	15.45	18.44	20.85	15.80	19.02	21.62
	(0.81)	(0.79)	(0.78)	(0.82)	(0.80)	(0.80)	(0.83)	(0.81)	(0.81)

Annual Std Dev = 0.90 Exercise Price = 40.

	Interest Rate = 0.05			Interest Rate = 0.10			Interest Rate = 0.15		
Price	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months	3 Months	6 Months	9 Months
28.	1.89	3.92	5.61	1.97	4.12	5.92	2.05	4.32	6.24
	(0.29)	(0.42)	(0.49)	(0.30)	(0.43)	(0.51)	(0.31)	(0.45)	(0.53)
32.	3.28	5.77	7.72	3.41	6.03	8.10	3.53	6.30	8.50
	(0.40)	(0.50)	(0.56)	(0.41)	(0.52)	(0.58)	(0.43)	(0.53)	(0.60)
36.	5.11	7.93	10.08	5.27	8.25	10.54	5.44	8.58	11.01
	(0.51)	(0.58)	(0.62)	(0.52)	(0.59)	(0.64)	(0.53)	(0.61)	(0.66)
40.	7.33	10.36	12.66	7.54	10.75	13.19	7.75	11.14	13.73
	(0.60)	(0.64)	(0.67)	(0.61)	(0.65)	(0.69)	(0.62)	(0.67)	(0.70)
44.	9.89	13.03	15.42	10.14	13.47	16.02	10.39	13.92	16.62
	(0.68)	(0.69)	(0.71)	(0.69)	(0.71)	(0.73)	(0.70)	(0.72)	(0.74)
48.	12.74	15.91	18.35	13.03	16.40	19.01	13.32	16.89	19.67
	(0.74)	(0.74)	(0.75)	(0.75)	(0.75)	(0.76)	(0.76)	(0.77)	(0.78)
52.	15.83	18.95	21.41	16.15	19.49	22.13	16.48	20.03	22.85
	(0.80)	(0.78)	(0.78)	(0.81)	(0.79)	(0.79)	(0.81)	(0.80)	(0.81)

TABLE 2. Option Values and Hedge Ratios for Different Stock Prices and Interest Rates, by Standard Deviation and Maturity.

Annual Std Dev = 0.20 Exercise Price = 40.

	3 Months			6 Months			9 Months		
Price	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	0.00	0.00	0.00	0.01	0.02	0.04	0.07	0.12	0.20
	(0.00)	(0.01)	(0.04)	(0.00)	(0.02)	(0.06)	(0.00)	(0.03)	(0.09)
32.	0.02	0.03	0.04	0.18	0.26	0.37	0.44	0.65	0.93
	(0.02)	(0.09)	(0.16)	(0.03)	(0.12)	(0.22)	(0.04)	(0.16)	(0.29)
36.	0.36	0.45	0.55	0.94	1.22	1.55	1.50	2.01	2.61
	(0.19)	(0.31)	(0.38)	(0.23)	(0.37)	(0.46)	(0.26)	(0.44)	(0.55)
40.	1.85	2.12	2.41	2.76	3.31	3.92	3.51	4.35	5.27
	(0.57)	(0.60)	(0.62)	(0.62)	(0.66)	(0.70)	(0.66)	(0.73)	(0.77)
44.	4.80	5.22	5.65	5.63	6.41	7.21	6.38	7.49	8.64
	(0.87)	(0.82)	(0.80)	(0.89)	(0.86)	(0.86)	(0.92)	(0.90)	(0.90)
48.	8.54	9.02	9.49	9.18	10.08	10.97	9.83	11.11	12.39
	(0.98)	(0.94)	(0.91)	(0.98)	(0.96)	(0.94)	(0.99)	(0.97)	(0.96)
52.	12.50	12.99	13.47	13.04	13.98	14.91	13.60	14.96	16.30
	(1.00)	(0.98)	(0.97)	(1.00)	(0.99)	(0.98)	(1.00)	(0.99)	(0.99)

Annual Std Dev = 0.30 Exercise Price = 40.

	3 Months			6 Months			9 Months		
Price	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	0.02	0.02	0.03	0.18	0.23	0.29	0.45	0.59	0.76
	(0.01)	(0.07)	(0.14)	(0.02)	(0.09)	(0.17)	(0.02)	(0.11)	(0.21)
32.	0.19	0.23	0.27	0.70	0.86	1.04	1.27	1.58	1.94
	(0.09)	(0.20)	(0.28)	(0.11)	(0.24)	(0.33)	(0.12)	(0.28)	(0.38)
36.	0.92	1.05	1.19	1.89	2.21	2.56	2.72	3.25	3.84
	(0.29)	(0.39)	(0.45)	(0.32)	(0.44)	(0.51)	(0.35)	(0.49)	(0.56)
40.	2.63	2.89	3.16	3.85	4.36	4.90	4.84	5.59	6.40
	(0.56)	(0.59)	(0.61)	(0.60)	(0.63)	(0.66)	(0.63)	(0.68)	(0.71)
44.	5.36	5.73	6.11	6.55	7.22	7.91	7.54	8.50	9.49
	(0.79)	(0.75)	(0.74)	(0.81)	(0.79)	(0.78)	(0.83)	(0.82)	(0.82)
48.	8.79	9.23	9.68	9.78	10.58	11.38	10.70	11.82	12.94
	(0.92)	(0.86)	(0.84)	(0.93)	(0.89)	(0.87)	(0.94)	(0.91)	(0.90)
52.	12.59	13.06	13.53	13.37	14.24	15.11	14.18	15.41	16.63
	(0.97)	(0.93)	(0.90)	(0.98)	(0.94)	(0.92)	(0.98)	(0.96)	(0.94)

Annual Std Dev = 0.40 Exercise Price = 40.

	3 Months			6 Months			9 Months		
Price	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	0.12	0.13	0.15	0.55	0.65	0.76	1.09	1.30	1.54
	(0.05)	(0.15)	(0.23)	(0.06)	(0.17)	(0.26)	(0.07)	(0.20)	(0.30)
32.	0.53	0.60	0.67	1.42	1.62	1.85	2.26	2.62	3.02
	(0.17)	(0.29)	(0.36)	(0.19)	(0.32)	(0.40)	(0.20)	(0.35)	(0.44)
36.	1.57	1.72	1.87	2.88	3.22	3.58	3.96	4.49	5.06
	(0.36)	(0.44)	(0.49)	(0.38)	(0.48)	(0.53)	(0.41)	(0.51)	(0.58)
40.	3.42	3.67	3.92	4.95	5.43	5.93	6.17	6.87	7.60
	(0.56)	(0.59)	(0.61)	(0.59)	(0.62)	(0.65)	(0.61)	(0.66)	(0.69)
44.	6.04	6.38	6.72	7.57	8.18	8.80	8.82	9.67	10.56
	(0.74)	(0.71)	(0.71)	(0.76)	(0.74)	(0.75)	(0.78)	(0.77)	(0.78)
48.	9.26	9.66	10.06	10.63	11.34	12.06	11.83	12.82	13.82
	(0.86)	(0.81)	(0.79)	(0.87)	(0.83)	(0.82)	(0.88)	(0.85)	(0.85)
52.	12.85	13.29	13.73	14.01	14.80	15.59	15.11	16.21	17.31
	(0.93)	(0.88)	(0.85)	(0.94)	(0.89)	(0.87)	(0.95)	(0.91)	(0.90)

Annual Std Dev = 0.50 Exercise Price = 40.

	3 Months			6 Months			9 Months		
Price	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	0.32	0.35	0.39	1.08	1.22	1.37	1.88	2.14	2.42
	(0.11)	(0.22)	(0.30)	(0.11)	(0.24)	(0.33)	(0.12)	(0.27)	(0.36)
32.	0.99	1.07	1.16	2.23	2.46	2.71	3.32	3.71	4.12
	(0.24)	(0.35)	(0.42)	(0.25)	(0.38)	(0.45)	(0.27)	(0.40)	(0.48)
36.	2.26	2.41	2.58	3.89	4.23	4.59	5.20	5.73	6.28
	(0.40)	(0.48)	(0.52)	(0.42)	(0.51)	(0.56)	(0.44)	(0.54)	(0.59)
40.	4.21	4.44	4.69	6.05	6.51	6.97	7.49	8.15	8.83
	(0.57)	(0.60)	(0.62)	(0.59)	(0.62)	(0.65)	(0.61)	(0.65)	(0.68)
44.	6.78	7.09	7.40	8.65	9.21	9.77	10.13	10.92	11.72
	(0.71)	(0.70)	(0.70)	(0.73)	(0.72)	(0.73)	(0.74)	(0.74)	(0.76)
48.	9.85	10.22	10.59	11.60	12.25	12.91	13.07	13.96	14.87
	(0.82)	(0.78)	(0.77)	(0.83)	(0.80)	(0.79)	(0.84)	(0.82)	(0.82)
52.	13.27	13.68	14.10	14.84	15.56	16.29	16.24	17.23	18.22
	(0.89)	(0.84)	(0.82)	(0.90)	(0.86)	(0.84)	(0.91)	(0.87)	(0.86)

TABLE 2. Option Values and Hedge Ratios for Different Stock Prices and Interest Rates, by Standard Deviation and Maturity. (Continued)

Annual Std Dev = 0.60 Exercise Price = 40.

Price	3 Months			6 Months			9 Months		
	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	0.62	0.67	0.72	1.72	1.88	2.05	2.76	3.05	3.35
	(0.16)	(0.28)	(0.36)	(0.17)	(0.30)	(0.39)	(0.18)	(0.33)	(0.42)
32.	1.51	1.61	1.71	3.09	3.33	3.59	4.41	4.81	5.22
	(0.29)	(0.40)	(0.46)	(0.30)	(0.42)	(0.49)	(0.32)	(0.45)	(0.52)
36.	2.96	3.12	3.29	4.91	5.25	5.60	6.44	6.95	7.48
	(0.44)	(0.51)	(0.55)	(0.45)	(0.53)	(0.58)	(0.47)	(0.56)	(0.61)
40.	4.99	5.22	5.45	7.14	7.58	8.02	8.81	9.43	10.07
	(0.58)	(0.61)	(0.63)	(0.59)	(0.63)	(0.66)	(0.61)	(0.65)	(0.68)
44.	7.54	7.83	8.13	9.74	10.26	10.79	11.46	12.19	12.93
	(0.69)	(0.69)	(0.70)	(0.71)	(0.71)	(0.72)	(0.72)	(0.73)	(0.75)
48.	10.52	10.86	11.20	12.64	13.24	13.84	14.37	15.19	16.02
	(0.79)	(0.76)	(0.75)	(0.80)	(0.78)	(0.77)	(0.81)	(0.79)	(0.80)
52.	13.81	14.20	14.58	15.79	16.45	17.12	17.47	18.38	19.28
	(0.86)	(0.81)	(0.80)	(0.87)	(0.83)	(0.82)	(0.87)	(0.84)	(0.84)

Annual Std Dev = 0.70 Exercise Price = 40.

Price	3 Months			6 Months			9 Months		
	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	0.99	1.05	1.12	2.42	2.60	2.78	3.69	3.99	4.31
	(0.21)	(0.34)	(0.41)	(0.22)	(0.35)	(0.44)	(0.23)	(0.37)	(0.46)
32.	2.08	2.19	2.30	3.97	4.23	4.49	5.51	5.91	6.33
	(0.33)	(0.44)	(0.50)	(0.35)	(0.46)	(0.52)	(0.36)	(0.48)	(0.55)
36.	3.67	3.84	4.01	5.92	6.26	6.60	7.67	8.16	8.67
	(0.46)	(0.53)	(0.58)	(0.48)	(0.55)	(0.60)	(0.49)	(0.57)	(0.62)
40.	5.77	6.00	6.22	8.23	8.64	9.07	10.11	10.70	11.30
	(0.58)	(0.62)	(0.64)	(0.60)	(0.64)	(0.67)	(0.61)	(0.66)	(0.69)
44.	8.32	8.59	8.87	10.84	11.33	11.83	12.79	13.47	14.16
	(0.69)	(0.69)	(0.70)	(0.70)	(0.71)	(0.72)	(0.71)	(0.72)	(0.74)
48.	11.23	11.55	11.88	13.72	14.27	14.83	15.69	16.45	17.21
	(0.77)	(0.75)	(0.75)	(0.78)	(0.76)	(0.77)	(0.79)	(0.78)	(0.79)
52.	14.44	14.80	15.16	16.81	17.42	18.04	18.76	19.59	20.43
	(0.83)	(0.80)	(0.79)	(0.84)	(0.81)	(0.80)	(0.85)	(0.82)	(0.82)

Annual Std Dev = 0.80 Exercise Price = 40.

Price	3 Months			6 Months			9 Months		
	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	1.42	1.49	1.57	3.16	3.35	3.55	4.64	4.95	5.27
	(0.25)	(0.38)	(0.45)	(0.26)	(0.40)	(0.48)	(0.27)	(0.41)	(0.50)
32.	2.67	2.79	2.91	4.87	5.13	5.40	6.62	7.01	7.42
	(0.37)	(0.47)	(0.53)	(0.38)	(0.49)	(0.55)	(0.40)	(0.51)	(0.57)
36.	4.39	4.56	4.73	6.93	7.26	7.60	8.88	9.36	9.85
	(0.49)	(0.56)	(0.60)	(0.50)	(0.57)	(0.62)	(0.51)	(0.59)	(0.64)
40.	6.55	6.77	6.99	9.30	9.70	10.10	11.39	11.95	12.52
	(0.59)	(0.63)	(0.66)	(0.60)	(0.64)	(0.68)	(0.62)	(0.66)	(0.69)
44.	9.10	9.36	9.63	11.94	12.40	12.87	14.11	14.75	15.39
	(0.68)	(0.69)	(0.70)	(0.69)	(0.71)	(0.72)	(0.70)	(0.72)	(0.74)
48.	11.98	12.28	12.59	14.81	15.33	15.85	17.02	17.73	18.44
	(0.75)	(0.74)	(0.75)	(0.76)	(0.76)	(0.76)	(0.77)	(0.77)	(0.78)
52.	15.11	15.45	15.80	17.86	18.44	19.02	20.08	20.85	21.62
	(0.81)	(0.79)	(0.78)	(0.82)	(0.80)	(0.80)	(0.83)	(0.81)	(0.81)

Annual Std Dev = 0.90 Exercise Price = 40.

Price	3 Months			6 Months			9 Months		
	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15	R=0.05	R=0.10	R=0.15
28.	1.89	1.97	2.05	3.92	4.12	4.32	5.61	5.92	6.24
	(0.29)	(0.42)	(0.49)	(0.30)	(0.43)	(0.51)	(0.31)	(0.45)	(0.53)
32.	3.28	3.41	3.53	5.77	6.03	6.30	7.72	8.10	8.50
	(0.40)	(0.50)	(0.56)	(0.41)	(0.52)	(0.58)	(0.43)	(0.53)	(0.60)
36.	5.11	5.27	5.44	7.93	8.25	8.58	10.08	10.54	11.01
	(0.51)	(0.58)	(0.62)	(0.52)	(0.59)	(0.64)	(0.53)	(0.61)	(0.66)
40.	7.33	7.54	7.75	10.36	10.75	11.14	12.66	13.19	13.73
	(0.60)	(0.64)	(0.67)	(0.61)	(0.65)	(0.69)	(0.62)	(0.67)	(0.70)
44.	9.89	10.14	10.39	13.03	13.47	13.92	15.42	16.02	16.62
	(0.68)	(0.69)	(0.71)	(0.69)	(0.71)	(0.73)	(0.70)	(0.72)	(0.74)
48.	12.74	13.03	13.32	15.91	16.40	16.89	18.35	19.01	19.67
	(0.74)	(0.74)	(0.75)	(0.75)	(0.75)	(0.76)	(0.76)	(0.77)	(0.78)
52.	15.83	16.15	16.48	18.95	19.49	20.03	21.41	22.13	22.85
	(0.80)	(0.78)	(0.78)	(0.81)	(0.79)	(0.79)	(0.81)	(0.80)	(0.81)

TABLE 3. Option Values and Hedge Ratios for Different Stock Prices and Standard Deviations, by Maturity and Interest Rate.

3 Months R = 0.05 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.00	0.02	0.12	0.32	0.62	0.99	1.42	1.89
	(0.00)	(0.01)	(0.05)	(0.11)	(0.16)	(0.21)	(0.25)	(0.29)
32.	0.02	0.19	0.53	0.99	1.51	2.08	2.67	3.28
	(0.02)	(0.09)	(0.17)	(0.24)	(0.29)	(0.33)	(0.37)	(0.40)
36.	0.36	0.92	1.57	2.26	2.96	3.67	4.39	5.11
	(0.19)	(0.29)	(0.36)	(0.40)	(0.44)	(0.46)	(0.49)	(0.51)
40.	1.85	2.63	3.42	4.21	4.99	5.77	6.55	7.33
	(0.57)	(0.56)	(0.56)	(0.57)	(0.58)	(0.58)	(0.59)	(0.60)
44.	4.80	5.36	6.04	6.78	7.54	8.32	9.10	9.89
	(0.87)	(0.79)	(0.74)	(0.71)	(0.69)	(0.69)	(0.68)	(0.68)
48.	8.54	8.79	9.26	9.85	10.52	11.23	11.98	12.74
	(0.98)	(0.92)	(0.86)	(0.82)	(0.79)	(0.77)	(0.75)	(0.74)
52.	12.50	12.59	12.85	13.27	13.81	14.44	15.11	15.83
	(1.00)	(0.97)	(0.93)	(0.89)	(0.86)	(0.83)	(0.81)	(0.80)

3 Months R = 0.10 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.00	0.02	0.13	0.35	0.67	1.05	1.49	1.97
	(0.00)	(0.02)	(0.06)	(0.11)	(0.17)	(0.22)	(0.26)	(0.30)
32.	0.03	0.23	0.60	1.07	1.61	2.19	2.79	3.41
	(0.03)	(0.11)	(0.19)	(0.25)	(0.30)	(0.35)	(0.38)	(0.41)
36.	0.45	1.05	1.72	2.41	3.12	3.84	4.56	5.27
	(0.23)	(0.32)	(0.38)	(0.42)	(0.45)	(0.48)	(0.50)	(0.52)
40.	2.12	2.89	3.67	4.44	5.22	6.00	6.77	7.54
	(0.62)	(0.60)	(0.59)	(0.59)	(0.59)	(0.60)	(0.60)	(0.61)
44.	5.22	5.73	6.38	7.09	7.83	8.59	9.36	10.14
	(0.89)	(0.81)	(0.76)	(0.73)	(0.71)	(0.70)	(0.69)	(0.69)
48.	9.02	9.23	9.66	10.22	10.86	11.55	12.28	13.03
	(0.98)	(0.93)	(0.87)	(0.83)	(0.80)	(0.78)	(0.76)	(0.75)
52.	12.99	13.06	13.29	13.68	14.20	14.80	15.45	16.15
	(1.00)	(0.98)	(0.94)	(0.90)	(0.87)	(0.84)	(0.82)	(0.81)

3 Months R = 0.15 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.00	0.03	0.15	0.39	0.72	1.12	1.57	2.05
	(0.00)	(0.02)	(0.07)	(0.12)	(0.18)	(0.23)	(0.27)	(0.31)
32.	0.04	0.27	0.67	1.16	1.71	2.30	2.91	3.53
	(0.04)	(0.12)	(0.20)	(0.27)	(0.32)	(0.36)	(0.40)	(0.43)
36.	0.55	1.19	1.87	2.58	3.29	4.01	4.73	5.44
	(0.26)	(0.35)	(0.41)	(0.44)	(0.47)	(0.49)	(0.51)	(0.53)
40.	2.41	3.16	3.92	4.69	5.45	6.22	6.99	7.75
	(0.66)	(0.63)	(0.61)	(0.61)	(0.61)	(0.61)	(0.62)	(0.62)
44.	5.65	6.11	6.72	7.40	8.13	8.87	9.63	10.39
	(0.92)	(0.83)	(0.78)	(0.74)	(0.72)	(0.71)	(0.70)	(0.70)
48.	9.49	9.68	10.06	10.59	11.20	11.88	12.59	13.32
	(0.99)	(0.94)	(0.88)	(0.84)	(0.81)	(0.79)	(0.77)	(0.76)
52.	13.47	13.53	13.73	14.10	14.58	15.16	15.80	16.48
	(1.00)	(0.98)	(0.95)	(0.91)	(0.87)	(0.85)	(0.83)	(0.81)

6 Months R = 0.05 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.01	0.18	0.55	1.08	1.72	2.42	3.16	3.92
	(0.01)	(0.07)	(0.15)	(0.22)	(0.28)	(0.34)	(0.38)	(0.42)
32.	0.18	0.70	1.42	2.23	3.09	3.97	4.87	5.77
	(0.09)	(0.20)	(0.29)	(0.35)	(0.40)	(0.44)	(0.47)	(0.50)
36.	0.94	1.89	2.88	3.89	4.91	5.92	6.93	7.93
	(0.31)	(0.39)	(0.44)	(0.48)	(0.51)	(0.53)	(0.56)	(0.58)
40.	2.76	3.85	4.95	6.05	7.14	8.23	9.30	10.36
	(0.60)	(0.59)	(0.59)	(0.60)	(0.61)	(0.62)	(0.63)	(0.64)
44.	5.63	6.55	7.57	8.65	9.74	10.84	11.94	13.03
	(0.82)	(0.75)	(0.71)	(0.70)	(0.69)	(0.69)	(0.69)	(0.69)
48.	9.18	9.78	10.63	11.60	12.64	13.72	14.81	15.91
	(0.94)	(0.86)	(0.81)	(0.78)	(0.76)	(0.75)	(0.74)	(0.74)
52.	13.04	13.37	14.01	14.84	15.79	16.81	17.86	18.95
	(0.98)	(0.93)	(0.88)	(0.84)	(0.81)	(0.80)	(0.79)	(0.78)

TABLE 3. Option Values and Hedge Ratios for Different Stock Prices and Standard Deviations, by Maturity and Interest Rate. (Continued)

6 Months R = 0.10 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.02	0.23	0.65	1.22	1.88	2.60	3.35	4.12
	(0.02)	(0.09)	(0.17)	(0.24)	(0.30)	(0.35)	(0.40)	(0.43)
32.	0.26	0.86	1.62	2.46	3.33	4.23	5.13	6.03
	(0.12)	(0.24)	(0.32)	(0.38)	(0.42)	(0.46)	(0.49)	(0.52)
36.	1.22	2.21	3.22	4.23	5.25	6.26	7.26	8.25
	(0.37)	(0.44)	(0.48)	(0.51)	(0.53)	(0.55)	(0.57)	(0.59)
40.	3.31	4.36	5.43	6.51	7.58	8.64	9.70	10.75
	(0.66)	(0.63)	(0.62)	(0.62)	(0.63)	(0.64)	(0.64)	(0.65)
44.	6.41	7.22	8.18	9.21	10.26	11.33	12.40	13.47
	(0.86)	(0.79)	(0.74)	(0.72)	(0.71)	(0.71)	(0.71)	(0.71)
48.	10.08	10.58	11.34	12.25	13.24	14.27	15.33	16.40
	(0.96)	(0.89)	(0.83)	(0.80)	(0.78)	(0.76)	(0.76)	(0.75)
52.	13.98	14.24	14.80	15.56	16.45	17.42	18.44	19.49
	(0.99)	(0.94)	(0.89)	(0.86)	(0.83)	(0.81)	(0.80)	(0.79)

6 Months R = 0.15 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.04	0.29	0.76	1.37	2.05	2.78	3.55	4.32
	(0.03)	(0.11)	(0.20)	(0.27)	(0.33)	(0.37)	(0.41)	(0.45)
32.	0.37	1.04	1.85	2.71	3.59	4.49	5.40	6.30
	(0.16)	(0.28)	(0.35)	(0.40)	(0.45)	(0.48)	(0.51)	(0.53)
36.	1.55	2.56	3.58	4.59	5.60	6.60	7.60	8.58
	(0.44)	(0.49)	(0.51)	(0.54)	(0.56)	(0.57)	(0.59)	(0.61)
40.	3.92	4.90	5.93	6.97	8.02	9.07	10.10	11.14
	(0.73)	(0.68)	(0.66)	(0.65)	(0.65)	(0.66)	(0.66)	(0.67)
44.	7.21	7.91	8.80	9.77	10.79	11.83	12.87	13.92
	(0.90)	(0.82)	(0.77)	(0.74)	(0.73)	(0.72)	(0.72)	(0.72)
48.	10.97	11.38	12.06	12.91	13.84	14.83	15.85	16.89
	(0.97)	(0.91)	(0.85)	(0.82)	(0.79)	(0.78)	(0.77)	(0.77)
52.	14.91	15.11	15.59	16.29	17.12	18.04	19.02	20.03
	(0.99)	(0.96)	(0.91)	(0.87)	(0.84)	(0.82)	(0.81)	(0.80)

9 Months R = 0.05 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.07	0.45	1.09	1.88	2.76	3.69	4.64	5.61
	(0.04)	(0.14)	(0.23)	(0.30)	(0.36)	(0.41)	(0.45)	(0.49)
32.	0.44	1.27	2.26	3.32	4.41	5.51	6.62	7.72
	(0.16)	(0.28)	(0.36)	(0.42)	(0.46)	(0.50)	(0.53)	(0.56)
36.	1.50	2.72	3.96	5.20	6.44	7.67	8.88	10.08
	(0.38)	(0.45)	(0.49)	(0.52)	(0.55)	(0.58)	(0.60)	(0.62)
40.	3.51	4.84	6.17	7.49	8.81	10.11	11.39	12.66
	(0.62)	(0.61)	(0.61)	(0.62)	(0.63)	(0.64)	(0.66)	(0.67)
44.	6.38	7.54	8.82	10.13	11.46	12.79	14.11	15.42
	(0.80)	(0.74)	(0.71)	(0.70)	(0.70)	(0.70)	(0.70)	(0.71)
48.	9.83	10.70	11.83	13.07	14.37	15.69	17.02	18.35
	(0.91)	(0.84)	(0.79)	(0.77)	(0.75)	(0.75)	(0.75)	(0.75)
52.	13.60	14.18	15.11	16.24	17.47	18.76	20.08	21.41
	(0.97)	(0.90)	(0.85)	(0.82)	(0.80)	(0.79)	(0.78)	(0.78)

9 Months R = 0.10 Exercise Price = 40.

Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.12	0.59	1.30	2.14	3.05	3.99	4.95	5.92
	(0.06)	(0.17)	(0.26)	(0.33)	(0.39)	(0.44)	(0.48)	(0.51)
32.	0.65	1.58	2.62	3.71	4.81	5.91	7.01	8.10
	(0.22)	(0.33)	(0.40)	(0.45)	(0.49)	(0.52)	(0.55)	(0.58)
36.	2.01	3.25	4.49	5.73	6.95	8.16	9.36	10.54
	(0.46)	(0.51)	(0.53)	(0.56)	(0.58)	(0.60)	(0.62)	(0.64)
40.	4.35	5.59	6.87	8.15	9.43	10.70	11.95	13.19
	(0.70)	(0.66)	(0.65)	(0.65)	(0.66)	(0.67)	(0.68)	(0.69)
44.	7.49	8.50	9.67	10.92	12.19	13.47	14.75	16.02
	(0.86)	(0.78)	(0.75)	(0.73)	(0.72)	(0.72)	(0.72)	(0.73)
48.	11.11	11.82	12.82	13.96	15.19	16.45	17.73	19.01
	(0.94)	(0.87)	(0.82)	(0.79)	(0.77)	(0.77)	(0.76)	(0.76)
52.	14.96	15.41	16.21	17.23	18.38	19.59	20.85	22.13
	(0.98)	(0.92)	(0.87)	(0.84)	(0.82)	(0.80)	(0.80)	(0.79)

TABLE 3. Option Values and Hedge Ratios for Different Stock Prices and Standard Deviations, by Maturity and Interest Rate. (Continued)

	9 Months R = 0.15 Exercise Price = 40.							
Price	S.D.=0.20	S.D.=0.30	S.D.=0.40	S.D.=0.50	S.D.=0.60	S.D.=0.70	S.D.=0.80	S.D.=0.90
28.	0.20	0.76	1.54	2.42	3.35	4.31	5.27	6.24
	(0.09)	(0.21)	(0.30)	(0.36)	(0.42)	(0.46)	(0.50)	(0.53)
32.	0.93	1.94	3.02	4.12	5.22	6.33	7.42	8.50
	(0.29)	(0.38)	(0.44)	(0.48)	(0.52)	(0.55)	(0.57)	(0.60)
36.	2.61	3.84	5.06	6.28	7.48	8.67	9.85	11.01
	(0.55)	(0.56)	(0.58)	(0.59)	(0.61)	(0.62)	(0.64)	(0.66)
40.	5.27	6.40	7.60	8.83	10.07	11.30	12.52	13.73
	(0.77)	(0.71)	(0.69)	(0.68)	(0.68)	(0.69)	(0.69)	(0.70)
44.	8.64	9.49	10.56	11.72	12.93	14.16	15.39	16.62
	(0.90)	(0.82)	(0.78)	(0.76)	(0.75)	(0.74)	(0.74)	(0.74)
48.	12.39	12.94	13.82	14.87	16.02	17.21	18.44	19.67
	(0.96)	(0.90)	(0.85)	(0.82)	(0.80)	(0.79)	(0.78)	(0.78)
52.	16.30	16.63	17.31	18.22	19.28	20.43	21.62	22.85
	(0.99)	(0.94)	(0.90)	(0.86)	(0.84)	(0.82)	(0.81)	(0.81)

Footnotes

1. For descriptions of options trading, see "New Game in Town" (1974), "Option Plays Are Spreading" (1973), and "The Values in Options" (1973) listed under References, and the current OCC *Prospectus*. Relevant books include Thorp and Kassouf (1967), and Malkiel and Quandt (1969).
2. See Black and Scholes (1972, 1973).
3. For some refinements and extensions, see Merton (1973).
4. More precisely, the position is short two-option contracts and long one round lot of stock, or long two-option contracts and short one round lot of stock.
5. The net money in a position is the value of any stock or options long in the investor's account minus the value of any stock or options that he is short. When he has no short positions, his net money is just the value of the stock or options in his account.
6. In fact, the $3,000 in certificates of deposit will earn interest, so the possible loss in the option position is even less than $1,000.
7. In this paragraph, I am using "speculative" and "conservative" as I take them to be used in ordinary language as related to investments.
8. Writing naked options is writing options without holding the underlying stock.
9. This assumes that the options are priced according to the formula. If they are overpriced, it may be possible to create a hedge that is long stock and short options and that shows losses only for large increases in the stock price.
10. For a proof of this, see Merton (1973, pp. 151-156).
11. This method is only an approximation. It assumes that the dividend is known for sure, and will neither be increased nor decreased. For options that expire in less than a year, this assumption gives a value that is very close to the correct value. When there is a possibility that the option will be exercised before an early ex-dividend date, we do calculations assuming expiration just before every ex-dividend date, and use the one that gives the highest value.
12. For extensive discussions of the role of information trading in determining a market maker's spread, see Bagehot (1971) and Black (1971).
13. For a discussion of the factors affecting the quality of the market for a stock, see Black (1971).

References

Bagehot, Walter. "The Only Game in Town." *Financial Analysts Journal* (March/April 1971), pp. 12-15.

Black, Fischer. "Toward a Fully Automated Exchange." *Financial Analysts Journal*, Parts I and II (July/August and November/December 1971).

Black, Fischer, and Myron Scholes. "The Valuation of Option Contracts and a Test of Market Efficiency." *Journal of Finance* (May 1972), pp. 399-417.

_________. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* (May/June 1973), pp. 637-654.

Malkiel, Burton G. and Richard E. Quandt. *Strategies and Rational Decisions in the Securities Options Market.* Cambridge, Massachusetts: MIT Press, 1969.

Merton, Robert G. "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* (Spring 1973), pp. 141-183.

"New Game in Town." *Wall Street Journal* (April 22, 1974), p. 1.

"Option Plays Are Spreading." *Business Week* (December 22, 1973), p. 104.

"The Values in Options." *Fortune* (November 1973), p. 89.

Thorp, Edward O. and Sheen T. Kassouf. *Beat the Market.* New York: Random House, 1967.