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Department of Statistics

Statistics C183/C283 Instructor: Nicolas Christou

Exam 2
17 May 2022

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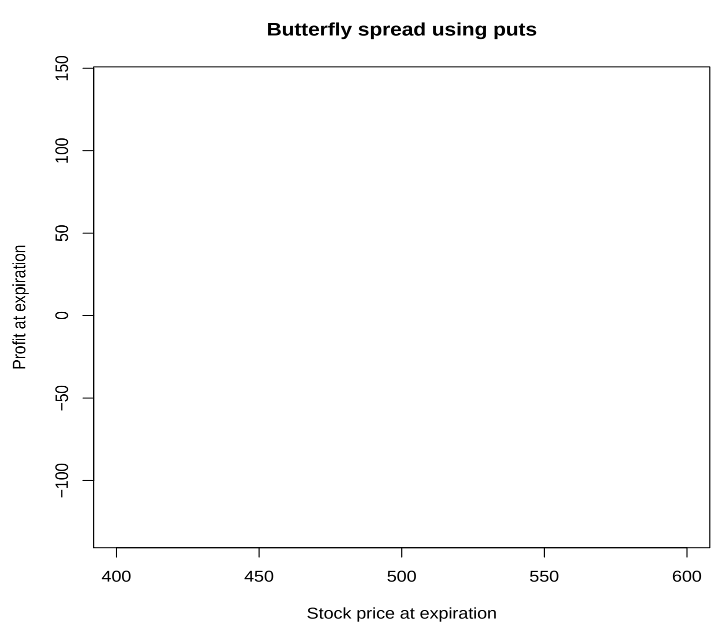
Answer the following questions:

1. Three put options on stock A have the same expiration date and exercise prices $\$450, \$500, \$550$ respectively. The value of these three put options are $\$24, \$68, \$115$. A butterfly spread can be created by buying one put with exercise price \$450, buying one put with exercise price \$550, and selling two puts each one with exercise price \$500. Construct the table showing the payoff for all the positions and for the total.

2. Refer to question (1). On the diagram below draw the profit for each position and the total (four graphs).



3. Consider the butterfly spread using call options. Let c_1, c_3 be the prices of the two long call options with exercise prices E_1, E_3 respectively and let c_2 be the price of the short call option with exercise price E_2. Note: For the butterfly spread using call options we buy two call options with exercise prices E_1, E_3 and we short two call options with exercise price E_2. In addition $E_3 > E_2 > E_1$ and $E_3 - E_2 = E_2 - E_1$. Show that $c_2 \leq \frac{1}{2}(c_1 + c_3)$.

4. Construct a reverse butterfly spread: Sell options with exercise prices E_1 and E_3 and buy two options with exercise price E_2, where E_2 is halfway between E_1 and E_3. Show the table of payoffs and the diagram.

5. Suppose the price of a European put option is \$2.50. The current stock price is $S_0 = \$47$, the time to expiration is 1 month, and the exercise price is $E = \$50$. The risk free interest rate is 6% per year with continuous compounding. Is there a riskless profit, and if yes what positions should the investor take?

6. Consider the multi-index model. The system that computes the $\Phi_i's$ can be written in vector and matrix form as $\boldsymbol{M\Phi} = \boldsymbol{R}$ and therefore, $\boldsymbol{\Phi} = \boldsymbol{M^{-1}R}$. Suppose you are working on a portfolio problem with five industries and 5 stocks in each industry. Write the elements of position (2,2) and position (3,1) of the matrix $\mathbf{M}$.

7. Consider the multigroup model with 5 industries and 6 stocks in each industry. Write an expression for the following:

   1. $\boldsymbol{\Phi_1}$.
   3. C_1^*.
   2. Z_1.
   4. Element of position (5,4) of the matrix $\mathbf{A}$ (where $\mathbf{A\Phi} = \boldsymbol{C}$).
   5. The first element of the vector $\mathbf{C}$.

8. Consider the following data on two stocks and assume that $R_f = 0.01$:

```
> cov(r)
       AAPL    IBM
AAPL 0.0058 0.0017
IBM  0.0017 0.0019

> cor(r)
      AAPL   IBM
AAPL 1.000 0.503
IBM  0.503 1.000

> colMeans(r)
      AAPL        IBM
0.03283572 0.01752981
```

Use the constant correlation model: Rank the stocks based on the excess return to standard deviation ratio, complete the table, and find the composition of the optimal portfolio when short sales are not allowed.

9. Consider the single index model. Show that the beta of the stock can be computed using the following three-step procedure:
   1. Obtain the residuals using the model $R_{it} = \alpha_0 + \epsilon_{it}$.
   2. Obtain the residuals using the model $R_{mt} = \gamma_0 + \delta_{mt}$.
   3. Regress the residuals from (1) on the residuals from (2) to get $\hat{\beta}$.

10. The price of the stock at $t = 0$ is $S_0 = \$80$. Suppose we know that in 4 months the stock will be either \$75 or \$85. The risk free interest rate is 5% with continuous compounding. Use no arbitrage arguments to find the price of a European put option that expires in 4 months with exercise price \$80.

11. Refer to question (10). Verify that risk neutral valuation for the European put option gives the same answer as in (10).

12. Refer to question (10). Using the same data find the price of a European call using no arbitrage arguments and using risk neutral valuation. Verify that the price of the European put from questions (10) and (11) and the price of the European call satisfy the put call parity relationship.

13. Provide a question that will result in the answers below:

    1. It is equal to $\mathrm{MAX}(S_T - E, 0)$.

    2. It is equal to $-\mathrm{MAX}(E - S_t, 0)$.

    3. It is equal to $-\mathrm{MAX}(S_T - E, 0)$.

    4. It is equal to $\mathrm{MAX}(E - S_T, 0)$.

    5. It is the relationship between the price of a European call and a European put written on the same underlying stock with the same expiration time and same exercise price.

14. Use put-call parity to show that the cost of a butterfly spread created from European puts is exactly the same as the cost of a butterfly spread created from European calls. Note: A butterfly spread using calls is created by buying one call with exercise price E_1, buy another call with exercise price E_3, and selling two calls with exercise E_2, where $E_1 < E_2 < E_3$ and $E_2 = \frac{E_1+E_3}{2}$. Similarly, using puts, a butterfly is created by buying one put with exercise price E_1, buy another put with exercise price E_3, and selling two puts with exercise E_2, where $E_1 < E_2 < E_3$ and $E_2 = \frac{E_1+E_3}{2}$. Also denote the cost of the three calls with c_1, c_2, c_3 and the cost of the three puts with p_1, p_2, p_3. We want to show that $c_1 + c_3 - 2c_2 = p_1 + p_3 - 2p_2$.

15. Using 30 stocks and the single index model no short sales allowed we obtained the following:

```
> a_no
             alpha      beta       rbar       sigma2
 [1,]  7 0.033549485 0.6831596 0.04067242 0.0097108478
 [2,] 24 0.043270972 0.9664530 0.05334765 0.0271679837
 [3,] 15 0.017778827 0.5557852 0.02357370 0.0013095130
 [4,] 29 0.027651835 2.0102884 0.04861202 0.1079108677
 [5,] 20 0.009917674 0.6798468 0.01700607 0.0019225044
 [6,] 26 0.012094519 0.9608449 0.02211273 0.0014863177
 [7,]  2 0.013399702 1.2063506 0.02597766 0.0029124353
 [8,]  1 0.013826091 1.4601222 0.02904999 0.0043276328
 [9,] 19 0.006693664 0.6595100 0.01357002 0.0230053998
[10,] 25 0.007582635 0.9791554 0.01779176 0.0029653518
[11,]  6 0.008329159 1.0911141 0.01970561 0.0055374960
[12,] 17 0.008040612 1.0607778 0.01910077 0.0007724195
[13,] 12 0.008141998 1.0887798 0.01949411 0.0012622593
           ratio       col1       col2       col3       col4
 [1,] 0.05807196  2.7909607   2.790961   48.06038   ????????
 [2,] 0.05416471  1.8621753   4.653136   34.37986   82.44024
 [3,] 0.04061586  9.5807586  14.233895  235.88710  318.32735
 [4,] 0.02368417  0.8869717  15.120866   37.44998  355.77732
 [5,] 0.02354364  ?????????  20.781021  240.41121  596.18853
 [6,] 0.02197309 13.6485324  34.429553  621.14775 1217.33628
 [7,] 0.02070514 10.3459187  44.775472  499.67865 1717.01494
 [8,] 0.01921071  9.4639304  54.239402  492.63812 2209.65306
 [9,] 0.01905963  0.3603524  54.599755   18.90658 2228.55964
[10,] 0.01714922  5.5446169  60.144372  323.31589 2551.87553
[11,] 0.01714359  3.6857738  63.830146  214.99429 2766.86982
[12,] 0.01706367 24.8581116  88.688257 1456.78550 4223.65531
[13,] 0.01698609 15.9523603 104.640618  939.14250 5162.79781
             col5  z_no_short   x_no_short
 [1,] 0.002000736   2.9545238  0.109049593
 [2,] 0.003255428   1.3549901  0.050011821
 [3,] 0.008547683  10.4158541  0.384442549
 [4,] 0.008880605   0.1417607  0.005232299
 [5,] 0.010694802   2.6412583  0.097487163
 [6,] 0.013426790   3.8131624  0.140741398
 [7,] 0.014613778   1.9180210  0.070792936
 [8,] 0.015250524   1.0581244  0.039054699
 [9,] 0.015270666   0.0855749  0.003158515
[10,] 0.015426450   0.3548540  0.013097436
[11,] 0.015516191   0.2106434  0.007774714
[12,] 0.015920881   1.3583715  0.050136628
[13,] ???????????   0.7862571  0.029020248
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We have also used $R_f = 0.001, \bar{R}_m = 0.01042646$, and $\sigma^2 = 0.0007424419$. Compute the cut-off rate C^* and the other two missing numbers in the table above. (There are three missing numbers.)