Name: SOLUTIONS

UCLA ID: _______________________

Answer the following questions:

a. Assume an investment of $100000 in asset A and $100000 in asset B. Suppose the daily volatilities for both assets are 1% and the correlation coefficient is 0.3. What is the 5-day 95% value at risk on this portfolio?

SD of daily changes in each asset is $1000

The variance of the portfolio change is $1000^2 + 1000^2 + 2 (0.3) (1000) (1000) = 2600000$

SD of portfolio's daily changes is $\sqrt{2600000} = 1612.45$

5-day 95% VAR is:

$1.645 \times 1612.45 \sqrt{5}$

b. Suppose your portfolio consists of 15 stocks. Let $(\Delta X_1, \Delta X_2, \ldots, \Delta X_{15})$, $i = 1, 2, \ldots, m$ be a random sample of $m$ vectors $\Delta X$ from $N_{15}(0, \Sigma)$. Find the distribution of

$\begin{pmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_{15} \end{pmatrix}$

$\sim N_{15}(0, \Sigma)$

USE MGF OF MULTIVARIATE NORMAL.
c. Suppose using spectral decomposition for the variance covariance matrix of 30 stocks we obtained the following eigenvalues:

```r
q <- eigen(cov_matrix)
q$values
[1] 0.1121619292 0.0580967552 0.0373882989 0.0355684463
[5] 0.0198080704 0.0181172754 0.0142485679 0.0082101040
[9] 0.0064204503 0.0054290295 0.0047297292 0.0045322445
[13] 0.003602019 0.0033294395 0.0031723272 0.0025223574
[17] 0.0019461610 0.0018166614 0.0017288426 0.0013777506
[21] 0.0011878728 0.0010810727 0.0006736017 0.0006310862
[25] 0.0005701274 0.0004971938 0.0004437374 0.0003608355
[29] 0.0003064061 0.0002221172
```

> sum(q$values)

```
[1] 0.3501776
```

How much is the total variance (sum of the variances of the 30 stocks?) What percent of the total variance is explained by the first 5 principal components? Compute the result and derive it theoretically.

\[
\text{TOTAL VARIANCE} = \sum \lambda_i = 0.3501776
\]

**FIRST 5 PRINCIPAL COMPONENTS**

\[
\frac{0.11 + 0.058 + 0.037 + 0.036 + 0.0195}{0.3501776} = 0.75
\]

\[
\approx 75\% 
\]

\[
\Sigma = \rho \Lambda \rho'
\]

\[
\text{var}(\Sigma) = \rho \text{var}(\Lambda) \rho' = \rho' \Lambda \rho = \sum_{i=1}^5 
\]

This is the **total variance**
d. Using 30 stocks and the single index model no short sales allowed we obtained the following:

\[
\begin{array}{c|c|c|c|c|c}
\text{col1} & \text{col2} & \text{col3} & \text{col4} & \text{col5} \\
\hline
\text{ratio} & \text{alpha} & \text{beta} & \text{rbar} & \text{sigma2} & \text{z_no_short} & \text{x_no_short} \\
\hline
1. & 0.05807196 & 2.7909607 & 2.790961 & 48.06038 & \text{????????} & \text{????????} \\
2. & 0.05416471 & 1.8621753 & 4.653136 & 34.37986 & 82.44024 & 20.781021 \\
3. & 0.04061586 & 9.5807586 & 14.233895 & 235.88710 & 318.32735 & 44.775472 \\
4. & 0.02368417 & 0.8869717 & 15.120866 & 37.44998 & 355.77732 & 44.775472 \\
5. & 0.02354364 & \text{????????} & 20.781021 & 240.41121 & 566.18853 & 240.41121 \\
6. & 0.02197309 & 13.6485324 & 34.429553 & 621.14775 & 1217.33628 & 621.14775 \\
7. & 0.02070514 & 10.3459187 & 44.775472 & 499.67686 & 1717.01494 & 499.67686 \\
8. & 0.01921071 & 9.4639304 & 54.239402 & 492.63812 & 2209.65306 & 492.63812 \\
9. & 0.01905963 & 0.3603524 & 54.599755 & 18.90658 & 2228.55964 & 18.90658 \\
10. & 0.01714922 & 6.5446169 & 60.144372 & 323.31589 & 2551.87553 & 323.31589 \\
11. & 0.01714359 & 3.6857738 & 63.830146 & 214.99429 & 2766.86982 & 214.99429 \\
12. & 0.01706367 & 24.8581116 & 88.68257 & 1456.78550 & 4223.65531 & 1456.78550 \\
13. & 0.01698609 & 15.9523603 & 104.640618 & 939.14250 & 5162.79781 & 939.14250 \\
\end{array}
\]

We have also used \( R_f = 0.001, R_m = 0.01042646, \) and \( \sigma^2 = 0.0007424410. \) Compute the cut-off rate \( C^* \) and the other two missing numbers in the table above. (There are three missing numbers.)

\[
C^* = 0.016074552 \\
\text{col1} = 5.6601546 \\
\text{col4} = 48.06038
\]

(\textbf{PLEASE HANDOUT #39 - SINGLE INDEX MODEL STEPS})
e. Provide a question that will result in the answers below:

1. It is equal to \( \text{MAX}(S_T - E, 0) \).

   **What is the payoff of the holder of a call?**

2. It is equal to \( -\text{MAX}(E - S_T, 0) \).

   **What is the payoff of the seller of a put?**

3. It is equal to \( -\text{MAX}(S_T - E, 0) \).

   **What is the payoff of the seller of a call?**

4. It is equal to \( \text{MAX}(E - S_T, 0) \).

   **What is the payoff of the holder of a put?**

5. It is the relationship between the price of a European call and a European put written on the same underlying stock with the same expiration time and same exercise price.

   **What is put-call parity?**

d. Consider the multi-index model with two indexes: The market index \( I_1 \) and the industry index \( I_2 \). The regression model is \( R_i = \alpha_i + \beta_1 I_{1t} + \beta_2 I_{2t} + \epsilon_t \). While this model can be estimated directly as it is using least squares, it is preferred to create a model where the indexes are orthogonal (uncorrelated). This would simplify the computation of risk (why?) and the selection of optimal portfolios. To create a model with orthogonal indexes we follow this procedure: We first regress the industry index \( I_2 \) on the market index \( I_1 \) and obtain the residuals from this regression. Finally, we regress the stock’s return on the market index \( I_1 \) and the residuals obtain from the first step. Explain why the two predictors in this model are orthogonal.

   **Why? If indexes are orthogonal then the variance of stock \( i \) is equal to the sum of variances (no covariance).**

   **The two predictors are orthogonal because the residuals of the first regression are uncorrelated with \( I_1 \).**
g. The following plot shows the expected return against beta of the market portfolio \( \bar{R}_M \) and a portfolio \( \bar{R}_A \) that you manage based on some model you chose.

It is given that \( \bar{R}_A = 0.06, \beta_A = 0.5, \bar{R}_M = 0.034, \sigma_M^2 = 0.015, R_f = 0.001 \). You are a portfolio manager and a client has given you a target risk \( \beta_T = 0.25 \). Compute the following components:

Return due to "investor's risk."

\[
\bar{R}_T = \bar{R}_F + (\bar{R}_M - \bar{R}_F) \beta_T = 0.001 + (0.034 - 0.001) \times 0.25 = 0.00925
\]

\[
\bar{R}_T - \bar{R}_F = 0.00925
\]

Return due to "manager's risk." This is the return due to the manager's choice of a different risk than the target.

\[
\bar{R}_A - \bar{R}_T = \left(0.001 + (0.034 - 0.001) \times 0.5 \right) - 0.00925
\]

\[
= 0.00825
\]

h. Consider the multi-index model as discussed in class. Derived the covariance between stocks that belong in the same industry and the covariance between stocks that belong in different industries. Please show the details.

\[
\sigma_{i,k} = \text{cov} \left[ \alpha_i + b_i I_j + \varepsilon_i, \alpha_k + b_k I_j + \varepsilon_k \right] = b_i b_k \sigma_j^5 = b_i b_k \left[ b_j \sigma_m^5 + \sigma_{i,j}^2 \right]
\]

\[
\sigma_{i,k} = \text{cov} \left[ \alpha_i + b_i I_j + \varepsilon_i, \alpha_k + b_k I_I + \varepsilon_k \right] = b_i b_k \text{cov} \left[ I_j, I_I \right]
\]

\[
= b_i b_k \sigma_{m}^5
\]
i. Consider the following two measures of portfolio performance: The Sharpe ratio and the differential excess return. Show graphically a situation of two portfolios A and B that are ranked as \( A > B \) using the Sharpe ratio but at the same time \( B > A \) using the differential excess return. Please explain why \( A > B \) and \( B > A \) for the respective measures of performance mentioned above.

\[ A > B \text{ using SHARPE RATIO} \]

\[ \text{But } \bar{R}_B - \bar{R}_A > \bar{R}_A - \bar{R}_B \]

j. Consider the following five portfolios.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Average annual return (%)</th>
<th>Standard deviation (%)</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14</td>
<td>6</td>
<td>1.5</td>
</tr>
<tr>
<td>B</td>
<td>22</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>8</td>
<td>1.0</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>6</td>
<td>0.5</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
<td>10</td>
<td>2.0</td>
</tr>
</tbody>
</table>

What is the Treynor measure for portfolio A if \( R_f = 3\% \)?

\[
\frac{\bar{R}_A - R_f}{\beta_A} = \frac{0.14 - 0.03}{1.5} = 0.073
\]

What is the differential excess return for portfolio B if \( \beta \) is used as the measure of risk?

\[
\bar{R}_B' = \bar{R}_f + (\bar{R}_B - \bar{R}_f) \frac{\beta}{\beta_B} = 0.03 + (0.13 - 0.03) \frac{0.5}{0.8} = 0.06 \rightarrow \bar{R}_B = \bar{R}_B' = 0.12 - 0.08 = 0.14
\]

Suppose \( R_m = 13\% \) and \( \sigma_m = 5\% \). What is the differential excess return for portfolio D if standard deviation is used as a measure of risk?

\[
\bar{R}_D' = \bar{R}_f + (\bar{R}_D - \bar{R}_f) \frac{\sigma}{\sigma_D} = 0.03 + (\frac{0.13 - 0.03}{0.05}) 0.06 = 0.11 \rightarrow \bar{R}_D = \bar{R}_D' = 0.10 - 0.15 = -0.05
\]

k. Consider the multigroup model with 5 industries and 6 stocks in each industry. Write an expression for the following:

1. \( \Phi_1 \): \[
\sum_{i=1}^{5} \Phi_i \sigma_i \left( \text{First Group} \right)
\]
2. \( \sigma_1 \): \[
\sum_{i=1}^{5} \Phi_i \phi_{i,1} \]
3. \( \alpha_1 \): \[
\frac{1}{5} \sum_{i=1}^{5} (\bar{R} - \bar{R}_f) _i - \sigma_i (1 - \sigma_{i,1})
\]
4. Element of position (5,4) of the matrix A (where \( A \Phi = C \)).
5. The first element of the vector C.

\[
\sum_{i=1}^{5} \Phi_i \phi_{i,1} \left( 5 \times 5 \right) \]

\[
\frac{5 \times \Phi_{5,4}}{1 - \Phi_{5,5}}
\]

\[
\frac{6 - \sum_{i=1}^{5} (\bar{R}_i - \bar{R}_f)}{\sum_{i=1}^{5} \sigma_i (1 - \sigma_{i,1})}
\]