**University of California, Los Angeles**
**Department of Statistics**

**Statistics C183/C283** **Instructor: Nicolas Christou**

**Homework 10**

**Exercise 1:**
Suppose the return of the underlying stock of a European call is equal to the risk-free interest rate. Show that the probability that a European call option will be exercised at time $T$ is equal to $\Phi(d_2)$. Assume lognormal property of stock prices. Also, time now is 0, therefore $\Delta t = T$.

**Exercise 2:**
Consider a 6 month put option on a stock with exercise price $E = \$32$. The current stock price is \$30 and over the next 6 months it is expected to rise to \$36 or fall to \$27. The risk-free continuous interest rate s 6%. What is the risk-neutral probability of the stock rising to \$36?

**Exercise 3:**
Consider an American call option on a stock. The following is given: $S_0 = \$50$, the time to expiration is 15 months, the risk-free interest rate is $r_f = 8\%$ per year, $E = \$55$, and $\sigma = 25\%$ per year. In addition, we know that the stock will pay dividends of \$1.50 in 4 months from now and another \$1.50 in 10 months from now. Show that it will never be optimal to exercise early on either of the two dividend dates. Calculate the price of the option.

**Exercise 4:**
Answer the following questions:

a. Let $c = S^{-\frac{2r}{\sigma^2}}$. Does $c$ satisfy the Black-Scholes-Merton differential equation?

b. Suppose the volatility for a stock goes to zero, i.e. $\sigma \to 0$. It means the stock is riskless and must earn the risk free interest rate. Therefore, at expiration time of a call option, $S_T = S_0 e^{rt}$. What is the value of the call option at time zero (now)?

c. What is the result obtained by the Black-Scholes-Merton model for the situation in (b)?

**Exercise 5:**
Determine the value of the following call using the Black-Scholes-Merton model. The stock's current price is \$95 with $\sigma = 0.6$. The call's exercise price is \$105, and it expires in 8 months from now. Assume that the continuously compounded riskless rate of interest is 0.08.