Answer the following questions:

a. The betas of 30 stocks were obtained using simple regression in two successive periods: 2008-12-31 to 2013-01-31 (period 1) and 2013-02-28 to 2017-03-31 (period 2). There are 49 months in each period. Suppose we use the unadjusted betas in the first period as predictions of the betas in the second period. We can then compute the prediction sum of squares (PRESS) to evaluate the performance of these unadjusted betas. The following information is obtained from these data:

\[ \sum_{i=1}^{30} P_i = 32.44349 \quad \text{Sum of the betas in period 1.} \]
\[ \sum_{i=1}^{30} A_i = 32.20206 \quad \text{Sum of the betas in period 2.} \]
\[ \sum_{i=1}^{30} P_i^2 = 51.70104 \quad \text{Sum of the squared betas in period 1.} \]
\[ \sum_{i=1}^{30} A_i^2 = 48.43207 \quad \text{Sum of the squared betas in period 2.} \]
\[ \sum_{i=1}^{30} (A_i - \hat{A})(\hat{A}_i - \bar{A}) = 4.299189 \quad \hat{A}_i \text{ are the fitted values of the simple regression of } A \text{ on } P. \]


\[ \text{PRESS} = (\bar{A} - \bar{P})^2 + (1 - R^2)S_A^2 + (1 - R^2)S_P^2 \]

\[ \bar{A} = \frac{32.44}{30} = 1.0814 \]
\[ \bar{P} = \frac{32.20206}{30} = 1.073402 \]
\[ S_A^2 = \frac{1}{30} \left( \frac{48.43701}{30} - \frac{32.20206}{30} \right) = 0.4579 \]
\[ S_P^2 = \frac{1}{30} \left( \frac{51.70104}{30} - \frac{32.44349}{30} \right) = 0.5538 \]

\[ \sum (A_i - \bar{A})(\hat{A}_i - \bar{A}) = \sum (A_i - \bar{A})^2 = 4.2992 = SS\text{R} \]
\[ SS\text{R} = \hat{\beta}_1 \cdot \sum (P_i - \bar{P})^2 \rightarrow \hat{\beta}_1 = \frac{SS\text{R}}{\sum (P_i - \bar{P})^2} = \frac{4.2992}{51.70 - 32.44} = 0.2587 \]
\[ \hat{\beta}_1 = \sqrt{0.5889} = 0.7508 \]

\[ R^2 = \frac{SS\text{R}}{SS\text{T}} = \frac{4.2992}{48.43 - 32.20} = 0.3129 \]
\[ R^2 = 1 - \frac{0.0003658 + 0.1337 + 0.3146}{0.4487} = 0.4487 \]
b. An investor has $900000 invested in a diversified portfolio. Subsequently the investor inherits ABC company stock worth $100000. His financial adviser provided him with the following forecast information:

<table>
<thead>
<tr>
<th></th>
<th>$ (monthly)</th>
<th>$ (monthly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio</td>
<td>0.67%</td>
<td>2.37%</td>
</tr>
<tr>
<td>ABC Company</td>
<td>1.25</td>
<td>2.95</td>
</tr>
</tbody>
</table>

The correlation coefficient between ABC company stock returns and the portfolio is 0.40.

Assume that the investor keeps the ABC company stock. Answer the following questions:

1. Calculate the expected return of the new portfolio which includes the ABC company stock.

\[
(0.90 \times 0.0067 + 0.1 \times 0.0125) = 0.00728
\]

2. Calculate the covariance between ABC company stock and the portfolio.

\[
\sigma = 0.4 \begin{pmatrix} 0.0237 \\ 0.0295 \end{pmatrix} \begin{pmatrix} 0.0237 \\ 0.0295 \end{pmatrix} = 0.002797
\]

3. Calculate the standard deviation of his new portfolio which includes the ABC company stock.

\[
SD = \sqrt{0.90^2 \times 0.0067^2 + 0.1^2 \times 0.0125^2 + 2 \times 0.90 \times 0.1 \times 0.00237 \times 0.0125}
\]

\[
= 0.02267
\]

c. Refer to question (b). If the investor sells the ABC company stock, he will invest the proceeds in risk-free government securities yielding 0.42% per month. Calculate the:

1. Expected return of the new portfolio which includes the government securities.

\[
(0.90 \times 0.0067 + 0.1 \times 0.0042) = 0.00645
\]

2. The standard deviation of his new portfolio which includes the government securities.

\[
SD = \sqrt{0.90^2 \times 0.0067^2} = 0.0213
\]
d. Suppose the single index model holds. The data below were obtained for the period 2014-01-31 to 2017-03-31:

ticker1 <- c("C", "AAPL", "XOM", "GSPC")
gr <- getReturns(ticker, start='2014-01-31', end='2017-03-31')

# Stock 1:
q1 <- lm(gr$R[,1] ~ gr$R[,4])
summary(q1)

Coefficients:

            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.003053   0.008714  -0.350    0.728
gr$R[, 4]   1.511946   0.287376   5.261   6.75e-06

Residual standard error: 0.05188 on 36 degrees of freedom

# Stock 2:
q2 <- lm(gr$R[,2] ~ gr$R[,4])
summary(q2)

Coefficients:

            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.01118    0.00861   1.298    0.202
gr$R[, 4]   1.42125    0.28396   5.005 1.48e-05 ***

Residual standard error: 0.05126 on 36 degrees of freedom

# Stock 3:
q3 <- lm(gr$R[,3] ~ gr$R[,4])
summary(q3)

Coefficients:

            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.004758   0.006170  -0.771    0.44567
gr$R[, 4]   0.672401   0.203484   3.304   0.00216 **

Residual standard error: 0.03673 on 36 degrees of freedom

Use the Vasicek's technique to adjust the beta of stock 3 for the next period. Assume the average beta is one with standard deviation 0.5.

\[
\tilde{\beta}_{3, \text{adj}} = \frac{\beta_3}{\sqrt{\text{Var}\beta} + \text{Var}\tilde{\beta}} = \frac{0.203484 \left(1 + \frac{0.05}{0.203484} \right)^{0.672401}}{0.5 + 0.203484} = 0.7189
\]

e. Refer to question (d). Suppose we form an equally weighted portfolio of the three stocks. What will be the nonsystematic standard deviation of this portfolio?

\[
\sqrt{\frac{1}{3} \left( \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right)} = \sqrt{\frac{1}{3} \left( 0.05184 + 0.50500 + 0.03673 \right)} = 0.0272
\]
f. Refer to question (d). Assume that the variance of the market is \( \sigma_m^2 = 0.001 \). Find the variance of a portfolio consisting of 60% stock 1 and 40% stock 2.

\[
\text{VAR}(0.60R_1 + 0.40R_2) = 0.60^2 \left(1.5114 \times 0.001 + 0.05182^2\right) \\
+ 0.40^2 \left(1.42125 \times 0.001 + 0.03126^2\right) \\
+ 2 \times 0.60 \times 0.40 \left(1.5114\right) \left(1.42125 \right) \times 0.001 = 0.00357
\]

g. Consider a portfolio consisting of \( n+1 \) assets: \( n \) risky assets and the \( (n+1) \)st asset is the risk-free asset with guaranteed return \( R_f \). When short sales allowed, the efficient frontier of all feasible portfolios which can be constructed from these \( n+1 \) assets is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected return \( E \) is determined by solving the problem

\[
\min \quad \frac{1}{2} \mathbf{x}' \Sigma \mathbf{x} \\
\text{subject to} \quad E = \mathbf{R}_f + (\mathbf{R} - \mathbf{R}_f \mathbf{1})' \mathbf{x}
\]

Definitions:

\( \mathbf{x} \) \quad Vector of the weight of the \( n \) risky assets.
\( \Sigma \) \quad Variance covariance matrix of the \( n \) risky assets.
\( \mathbf{R} \) \quad Vector of the expected returns of the \( n \) risky assets.
\( \mathbf{1} = (1,1,\ldots,1)' \) \quad \( n \times 1 \) vector of ones.
\( R_f \) \quad Risk free rate.
\( E \) \quad Required expected return (combination of the \( n \) risky assets and \( R_f \)).

We showed in class that the weights of the \( n \) risky assets are given by

\[
\mathbf{x} = \frac{E - R_f}{(\mathbf{R} - R_f \mathbf{1})' \Sigma^{-1}(\mathbf{R} - R_f \mathbf{1})} \Sigma^{-1}(\mathbf{R} - R_f \mathbf{1}).
\]  

Consider now the point of tangency \( G \) (see for example handout #15). The composition of portfolio \( G \) as discussed in the handout is computed by finding first \( \mathbf{Z} = \Sigma^{-1}(\mathbf{R} - R_f \mathbf{1}) \). Show that when the required expected return \( E \) is equal to the expected return of portfolio \( G \) then using (1) the weights are exactly the same as the ones obtained using handout #15.

\[
\text{USE} \quad E = \mathbf{x}' \bar{\mathbf{R}} \quad \text{WHERE} \quad \mathbf{x} \quad \text{COMPOSITION OF} \\
\text{POINT OF TANGENCY}
\]

\[
\text{SINCE} \quad \mathbf{Z} = \Sigma^{-1} \mathbf{R} \quad \text{WE HAVE} \\
\mathbf{x} = \Sigma^{-1} \mathbf{R} \\
\]

\[
\text{WHERE} \quad \mathbf{R} = \bar{\mathbf{R}} - R_f \mathbf{1}
\]