Exercise 1
Refer to homework 1. Answer the following questions.


b. Suppose the investor has access to the risk free asset. Choose a value of $R_f$ to find the composition of the point of tangency $G$.

c. Refer to part (b). Create a graph similar to the following:

![Graph](image.png)

Note: The green dots represent the 5 stocks, the blue dot represent the point of tangency, and the other dots represent arbitrary portfolios using the 5 stocks.

Exercise 2
Please answer the following questions.

1. The mean returns and variance covariance matrix of the returns of three stocks ($C$, XOM, AAPL and the market SP500) are given below:

   **Mean returns:**
   
   $C$  XOM  AAPL  ~GSPC  
   0.005174  0.010617  0.016947  0.010846

   **Variance-covariance matrix:**
   
   $C$  $C$  XOM  AAPL  ~GSPC  
   XOM  0.010025  0.000000  0.000000  0.000000
   AAPL  0.000000  0.002123  0.000000  0.000000
   ~GSPC  0.000000  0.000000  0.005775  0.000000

   Assume short sales are allowed. Compute the composition of the minimum risk portfolio using only the three stocks (do not use the SP500).

2. Refer to questions (1). Assume $R_f = 0.001$ and short sales allowed. Find the composition of the optimal portfolio (point of tangency).
Exercise 3
Consider two stocks $A$ and $B$ with expected returns $\bar{R}_1, \bar{R}_2$, variances $\sigma_1^2, \sigma_2^2$, and covariance $\sigma_{12}$. Suppose short sales are allowed and risk free asset $R_f$ exists. Show that the composition of the optimal portfolio is

$$x_1 = \frac{\bar{R}_A \times \sigma_2^2 - \bar{R}_B \times \sigma_{12}}{R_A \times \sigma_2^2 + R_B \times \sigma_1^2 - (R_A + R_B) \times \sigma_{12}}$$

$$x_2 = 1 - x_1$$

Note: $\bar{R}_A = \bar{R}_1 - R_f$ and $\bar{R}_B = \bar{R}_2 - R_f$.

Exercise 4
Given the following:

<table>
<thead>
<tr>
<th>Stock</th>
<th>$R$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>0.12</td>
<td>0.20</td>
</tr>
<tr>
<td>Stock B</td>
<td>???</td>
<td>0.08</td>
</tr>
</tbody>
</table>

It is also given that $\rho_{AB} = 0.1$.

a. What expected return on stock $B$ would result in an optimum portfolio of $\frac{1}{2}A$ and $\frac{1}{2}B$? Assume short sales are allowed and that $R_f = 0.04$.

b. What expected return on stock $B$ would mean that stock $B$ would not be held? Assume short sales are allowed and that $R_f = 0.04$. 


Exercise 5
Please answer the following questions.

1. An investor wants to hold portfolio A as shown on the plot below. Suppose that short sales are allowed and \( R_f = 0.001 \). Suggest a better investing strategy. Note: Point G on the plot is the point of tangency.

2. Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz? Please explain your answer.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected return (%)</th>
<th>Standard deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>15</td>
<td>36</td>
</tr>
<tr>
<td>X</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Y</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Z</td>
<td>9</td>
<td>21</td>
</tr>
</tbody>
</table>

3. Suppose all stocks have \( E(R) = 15\% \), \( \sigma = 60\% \), and common correlation coefficient \( \rho = 0.5 \). What are the expected return and standard deviation of an equally weighted portfolio of \( n = 25 \) stocks?

4. Refer to question (3). What is the smallest number of stocks necessary to generate a portfolio with standard deviation of at most 43%?

5. Refer to question (3). As \( n \) gets larger, is it true that \( \sigma_p = \sigma \sqrt{\rho} \)? Please explain your answer.