Exercise 1
Refer to the lecture material on Friday, 04/02. In order to find the Lagrange multipliers $\lambda_1$ and $\lambda_2$ we must invert the matrix \[
\begin{pmatrix}
B & A \\
A & C
\end{pmatrix},
\]
where $A = 1'\Sigma^{-1}\bar{R}$, $B = \bar{R}'\Sigma^{-1}\bar{R}$, and $C = 1'\Sigma^{-1}1$. Show that $BC - A^2 > 0$. Note: Begin with $(AR - B1)'\Sigma^{-1}(AR - B1) > 0$ because $\Sigma$ is positive definite matrix.

Exercise 2
In the paper “An Analytic Derivation of the Efficient Portfolio Frontier,” The Journal of Financial and Quantitative Analysis, Vol. 7, No. 4, Robert Merton gives on page 1854 the proportion of the $k$th risky asset held in the frontier portfolio with expected return $E$ by
\[
x_k = E\frac{\sum_{j=1}^{m} v_{kj}(CE_j - A) + \sum_{j=1}^{m} v_{kj}(B - AE_j)}{D}, \quad k = 1, \ldots, m.
\]
Prove equation (1).

On the same page, it is shown that the expected return of the minimum risk portfolio is $\bar{E} = \frac{A}{D}$. Using equation (1) above show that the proportion of the $k$th risky asset of the minimum risk portfolio is $x_k = \frac{\sum_{j=1}^{m} v_{kj}}{D}, \quad k = 1, \ldots, m$.

Exercise 3
Find an expression of the correlation coefficient of two portfolios on the efficient frontier. See homework 1 for the covariance between two portfolios.

Exercise 4
The covariance matrix $Q$ of the returns of two stocks has the following inverse:
\[
> \text{solve}(Q)
\]
\[
[1,] \ 166.21139 \ -22.40241
[2,] \ -22.40241 \ 220.41076
\]
Answer the following questions:

a. Find the composition of the minimum risk portfolio.

b. It is given that the minimum risk portfolio (point A on the graph below) has standard deviation equal to 0.05408825 and expected return equal to 0.01315856. Portfolio B (see graph below) has expected return equal to 0.01219724. What is the composition of portfolio B in terms of portfolio A and the risk free asset? Assume $R_f = 0.011$.

c. The standard deviation of portfolio B is equal to 0.03. Given this level of risk, can you do better than the expected return of portfolio B? Please explain.

Exercise 5
Show that two portfolios on the capital allocation line are perfectly correlated.