University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 2

Exercise 1
Refer to the lecture material on Friday, 04/02. In order to find the Lagrange multipliers λ_1 and λ_2 we must invert the matrix $\begin{pmatrix} B & A \\ A & C \end{pmatrix}$, where $A = \mathbf{1}'\mathbf{\Sigma}^{-1}\bar{\mathbf{R}}$, $B = \bar{\mathbf{R}}'\mathbf{\Sigma}^{-1}\bar{\mathbf{R}}$, and $C = \mathbf{1}'\mathbf{\Sigma}^{-1}\mathbf{1}$. Show that $BC - A^2 > 0$. Note: Begin with $(A\bar{\mathbf{R}} - B\mathbf{1})'\mathbf{\Sigma^{-1}}(A\bar{\mathbf{R}} - B\mathbf{1}) > 0$ because $\mathbf{\Sigma}$ is positive definite matrix.

Exercise 2
In the paper "An Analytic Derivation of the Efficient Portfolio Frontier," *The Journal of Financial and Quantitative Analysis*, Vol. 7, No. 4, Robert Merton gives on page 1854 the proportion of the k_{th} risky asset held in the frontier portfolio with expected return E by

$$x_k = \frac{E\sum_{j=1}^m v_{kj}(CE_j - A) + \sum_{j=1}^m v_{kj}(B - AE_j)}{D}, \quad k = 1, \ldots, m. \tag{1}$$

Prove equation (1).

On the same page, it is shown that the expected return of the minimum risk portfolio is $\bar{E} = \frac{A}{C}$. Using equation (1) above show that the proportion of the k_{th} risky asset of the minimum risk portfolio is $x_k = \frac{\sum_{j=1}^m v_{kj}}{C}$, $k = 1, \ldots, m$.

Exercise 3
Find an expression of the correlation coefficient of two portfolios on the efficient frontier. See homework 1 for the covariance between two portfolios.

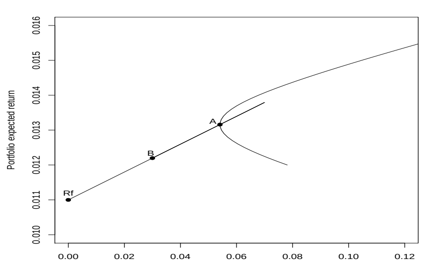
Exercise 4
The covariance matrix $\mathbf{Q}$ of the returns of two stocks has the following inverse:

```
> solve(Q)
          [,1]      [,2]
[1,] 166.21139 -22.40241
[2,] -22.40241 220.41076
```

Answer the following questions:

a. Find the composition of the minimum risk portfolio.

b. It is given that the minimum risk portfolio (point A on the graph below) has standard deviation equal to 0.05408825 and expected return equal to 0.01315856. Portfolio B (see graph below) has expected return equal to 0.01219724. What is the composition of portfolio B in terms of portfolio A and the risk free asset? Assume $R_f = 0.011$.



c. The standard deviation of portfolio B is equal to 0.03. Given this level of risk, can you do better than the expected return of portfolio B? Please explain.

Exercise 5
Access the following data:

`http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt.`

In `R` you can access the data from the command line as follows:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt", header=T)
```

Note: The most recent data are at the beginning of this data set, therefore you will need to adjust the following to get the returns:

```
r <- (a[-1,3:ncol(a)]-a[-nrow(a),3:ncol(a)])/a[-nrow(a),3:ncol(a)]
```

The previous command works when the most recent data are at the end of the data set (as in yopur project data for example).

In this data set you are given the close monthly prices from January 1986 to December 2003. The first column is the date and P_1, P_2, P_3, P_4, P_5 represent the close monthly prices for the stocks Exxon-Mobil, General Motors, Hewlett Packard, McDonalds, and Boeing respectively.

a. Convert the prices into returns for all the 5 stocks.

b. Compute and print the mean return for each stock and the variance-covariance matrix.

c. Use only Exxon-Mobil and Boeing stocks: For these 2 stocks find the composition, expected return, and standard deviation of the minimum risk portfolio.

d. Plot and print the portfolio possibilities curve and identify the efficient frontier on it (no short sales).

e. Use only Exxon-Mobil, McDonalds and Boeing stocks and assume short sales are allowed to answer the following question: For these 3 stocks compute the expected return and standard deviation for many combinations of x_a, x_b, x_c with $x_a + x_b + x_c = 1$ and plot and print the cloud of points. Reminder: Short sales are allowed. In case you need many combinations you can get them from here:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/datac183c283/statc183c283_abc.txt", header=T)
```

f. Assume $R_f = 0.001$ and that short sales are allowed. Find the composition, expected return and standard deviation of the portfolio of the point of tangency G and draw the tangent to the efficient frontier of question (e). Print the graph.

g. Find the expected return and standard deviation of the portfolio that consists of 60% G 40% risk free asset. Show this position on the capital allocation line (CAL).

h. Now assume that short sales allowed but risk free asset does not exist.

   1. Using $R_{f1} = 0.001$ and $R_{f2} = 0.002$ find the composition of two portfolios A and B (tangent to the efficient frontier - you found the one with $R_{f1} = 0.001$ in exercise f).
   2. Compute the covariance between portfolios A and B?
   3. Use your answers to (1) and (2) to trace out the efficient frontier of the stocks Exxon-Mobil, McDonalds, Boeing. Submit a printout of the graph that shows this efficient frontier on top of the cloud of points from question (e).
   4. Find the composition of the minimum risk portfolio (how much of each stock), its expected return, and standard deviation.