

Homework 3

Exercise 1

Consider two stocks A and B with expected returns \bar{R}_1, \bar{R}_2 , variances σ_1^2, σ_2^2 , and covariance σ_{12} . Suppose short sales are allowed and risk free asset R_f exists. Show that the composition of the optimal portfolio is

$$\begin{aligned} x_1 &= \frac{\bar{R}_A \times \sigma_2^2 - \bar{R}_B \times \sigma_{12}}{\bar{R}_A \times \sigma_2^2 + \bar{R}_B \times \sigma_1^2 - (\bar{R}_A + \bar{R}_B) \times \sigma_{12}} \\ x_2 &= 1 - x_1 \end{aligned}$$

Note: $\bar{R}_A = \bar{R}_1 - R_f$ and $\bar{R}_B = \bar{R}_2 - R_f$.

Exercise 2

Given the following:

Stock	\bar{R}	σ
Stock A	0.12	0.20
Stock B	???	0.08

It is also given that $\rho_{AB} = 0.1$.

- a. What expected return on stock B would result in an optimum portfolio of $\frac{1}{2}A$ and $\frac{1}{2}B$? Assume short sales are allowed and that $R_f = 0.04$.
- b. What expected return on stock B would mean that stock B would not be held? Assume short sales are allowed and that $R_f = 0.04$.

Exercise 3

Use a numerical example of three stocks with a value of R_f of your choice to find the point of tangency G and then (1) combine G with R_f to find portfolio A on CAL and (2) verify that A can be obtained by using the formula for the weights \mathbf{X} when the investor requires $\sum_{i=1}^n (\bar{R}_i - R_f)x_i + R_f = E$, where E is the expected value of portfolio A .

Exercise 4

Answer the following questions:

- a. An investor has \$900000 invested in a diversified portfolio. Subsequently the investor inherits ABC company stock worth \$100000. His financial adviser provided him with the following forecast information:

	\bar{R} (monthly)	σ (monthly)
Portfolio	0.67%	2.37%
ABC Compnay	1.25%	2.95%

The correlation coefficient between ABC company stock returns and the portfolio is 0.40.

Assume that the investor keeps the ABC company stock. Answer the following questions:

1. Calculate the expected return of the new portfolio which includes the ABC company stock.
 2. Calculate the covariance between ABC company stock and the portfolio.
 3. Calculate the standard deviation of his new portfolio which includes the ABC company stock.
- b. Refer to question (a). If the investor sells the ABC company stock, he will invest the proceeds in risk-free government securities yielding 0.42% per month. Calculate the:
 1. Expected return of the new portfolio which includes the government securities.
 2. The standard deviation of his new portfolio which includes the government securities.

Exercise 5

Answer the following questions:

- a. Consider a portfolio consisting of n risky assets. When short sales allowed, the efficient frontier of all feasible portfolios which can be constructed from these n assets is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected return E is determined by solving the problem

$$\begin{aligned} \min \quad & \frac{1}{2} \mathbf{x}' \boldsymbol{\Sigma} \mathbf{x} \\ \text{subject to} \quad & \bar{\mathbf{R}}' \mathbf{x} = E \\ \text{and} \quad & \mathbf{1}' \mathbf{x} = 1 \end{aligned}$$

Show that the weights of the optimal portfolio \mathbf{x} is given by $\mathbf{x} = \mathbf{g} + \mathbf{h}E$, where \mathbf{g} and \mathbf{h} are $n \times 1$ vectors, given by

$$\begin{aligned} \mathbf{g} &= \frac{1}{D} [B\boldsymbol{\Sigma}^{-1}\mathbf{1} - A\boldsymbol{\Sigma}^{-1}\bar{\mathbf{R}}] \\ \mathbf{h} &= \frac{1}{D} [C\boldsymbol{\Sigma}^{-1}\bar{\mathbf{R}} - A\boldsymbol{\Sigma}^{-1}\mathbf{1}]. \end{aligned}$$

The scalars A, B, C, D are defined as in the paper “An Analytic Derivation of the Efficient Portfolio Frontier,” by Robert Merton.

- b. Refer to question (a). Consider two portfolios a, b on the efficient frontier (other than the minimum risk portfolio). Show that the covariance between the two portfolios is given by

$$\text{cov}(R_a, R_b) = \frac{C}{D} \left(E_a - \frac{A}{C} \right) \left(E_b - \frac{A}{C} \right) + \frac{1}{C}.$$