Exercise 1

The solution is based on $Z = \Sigma^{-1} R$, where, $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix}$, and $R = \begin{pmatrix} \bar{R}_A = \bar{R}_1 - R_f \\ \bar{R}_B = \bar{R}_2 - R_f \end{pmatrix}$.

The inverse of $\Sigma$ is $\Sigma^{-1} = \frac{1}{\sigma_1^2 \sigma_2^2 (1-\rho^2)} \begin{pmatrix} \sigma_2^2 & -\rho \sigma_1 \sigma_2 \\ -\rho \sigma_1 \sigma_2 & \sigma_1^2 \end{pmatrix} \begin{pmatrix} \bar{R}_A \\ \bar{R}_B \end{pmatrix}$.

Therefore, $Z = \frac{1}{\sigma_1^2 \sigma_2^2 (1-\rho^2)} \begin{pmatrix} \sigma_2^2 & -\rho \sigma_1 \sigma_2 \\ -\rho \sigma_1 \sigma_2 & \sigma_1^2 \end{pmatrix} \begin{pmatrix} \bar{R}_A \\ \bar{R}_B \end{pmatrix}$.

It follows that $Z_1 = \frac{\bar{R}_A \sigma_2^2 - \bar{R}_B \rho \sigma_1 \sigma_2}{\sigma_1^2 \sigma_2^2 (1-\rho^2)}$ and $Z_2 = \frac{-\bar{R}_A \rho \sigma_1 \sigma_2 + \bar{R}_B \sigma_2^2}{\sigma_1^2 \sigma_2^2 (1-\rho^2)}$.

Finally, $X_1 = \frac{Z_1}{Z_1 + Z_2} = \frac{\bar{R}_A \sigma_2^2 - \bar{R}_B \rho \sigma_1 \sigma_2}{\bar{R}_A \sigma_1^2 + \bar{R}_B \sigma_2^2 - (\bar{R}_A + \bar{R}_B) \rho \sigma_1 \sigma_2}$, or $X_1 = \frac{\bar{R}_A \sigma_2^2 - \bar{R}_B \rho \sigma_1 \sigma_2}{\bar{R}_A \sigma_1^2 + \bar{R}_B \sigma_2^2 - (\bar{R}_A + \bar{R}_B) \rho \sigma_1 \sigma_2}$, and $X_2 = 1 - X_1$.

Exercise 2

The solution is based on finding the $Z$ values. Note: You can also solve this problem using the results of exercise 2.

a. Since $X_A = X_B = \frac{1}{2}$ it follows that $Z_A = Z_B$. We get the following system:

\[
\begin{align*}
0.12 - 0.04 &= 0.04 Z_A + 0.0016 Z_B \\
\bar{R}_B - 0.04 &= 0.0016 Z_A + 0.0064 Z_B
\end{align*}
\]

Solve for $Z_A$ to get $Z_A = 1.9231$. Therefore, $\bar{R}_B = 0.04 + (0.0016 + 0.0064)1.9231 \Rightarrow \bar{R}_B = 0.055385$.

b. Stock $B$ will not be held implies that $X_B = 0$ therefore, $Z_B = 0$.

\[
\begin{align*}
0.12 - 0.04 &= 0.04 Z_A + 0.0016 Z_B \\
\bar{R}_B - 0.04 &= 0.0016 Z_A + 0.0064 Z_B
\end{align*}
\]

But, $Z_B = 0$, therefore, $Z_A = 2$. It follows that $\bar{R}_B = 0.0432$. 

Exercise 3

\[ X = \frac{(E-R_F) \sum [\bar{R} - R_F]}{\left( \sum [\bar{R} - R_F] \right)^{\frac{1}{2}} \left( \sum \left( R - R_F \right) \right)} \]  \hspace{1cm} (1)

Substitute E with the expected return of the tangency point portfolio.

To find the composition of G, we use:

\[ X_G = \frac{\overline{G}}{\begin{array}{c} \sum \left( \bar{R} - R_F \right) \end{array}} \]

where

\[ \overline{G} = \frac{\sum \left( \bar{R} - R_F \right)}{\begin{array}{c} \sum \left( \bar{R} - R_F \right) \end{array}} \]

So,

\[ X_G = \frac{\sum \left( \bar{R} - R_F \right)}{\begin{array}{c} \sum \left( \bar{R} - R_F \right) \end{array}} \]

Replace E with \( E_G \) in (1) to show that \( X \) is the same as \( X_G \).
\[ X = \frac{(\bar{R} - \bar{R}_C)}{\sum' \frac{\bar{C}}{\bar{R}_C}} - \bar{R}_C \]

\[ = \frac{\sum' \bar{R}}{\sum' \bar{C}} - \bar{R}_C \]

\[ \approx \frac{\sum' \bar{R} - \bar{R}_C}{\sum' \bar{C}} \]

\[ = \frac{\sum' (\bar{R} - \bar{R}_C)}{\sum' \bar{C}} = X_0. \]
Exercise 4

\[ R_{p_1} = X_1' R + (1 - \frac{1}{n}) \tilde{R} \]

\[ R_{p_2} = X_2' R + (1 - \frac{1}{n}) \tilde{R} \]

\[
\text{Cor} (R_{p_1}, R_{p_2}) = \frac{\text{Cov} (R_{p_1}, R_{p_2})}{\sqrt{\text{Var} (R_{p_1})} \sqrt{\text{Var} (R_{p_2})}}
\]

\[
\text{Cov} (X_1' R + \text{const}, X_2' R + \text{const})
\]

\[
= \frac{\text{Cov} (X_1, X_2)}{\sqrt{\text{Var} (X_1' R + \text{const})} \sqrt{\text{Var} (X_2' R + \text{const})}}
\]

\[
= \frac{X_1' \Sigma X_2}{\sqrt{X_1' \Sigma X_1} \sqrt{X_2' \Sigma X_2}} = 1.
\]

Where, \( X_1 = \frac{(E_i - \bar{R}) \Sigma (\tilde{R} - \bar{R})!}{(\tilde{R} - \bar{R})' \Sigma (\tilde{R} - \bar{R})!} \) and \( X_2 = \frac{(E_j - \bar{R}) \Sigma (\tilde{R} - \bar{R})!}{(\tilde{R} - \bar{R})' \Sigma (\tilde{R} - \bar{R})!} \).
Exercise 3:

(a). 
1. \(0.90 \times 0.0067 + 0.1 \times 0.0225 = 0.0078\)
2. \(\text{Cov} = 0.4 \times (0.0227) \times (0.0275) = 0.000279\)
3. \(\text{SD} = \sqrt{0.9^2 (0.0227)^2 + 0.1^2 (0.0225)^2 + 2 \times 0.9 \times 0.1 \times 0.000279} = 0.0226\)

(b). 
1. \(0.90 \times 0.0067 + 0.1 \times (0.042) = 0.00645\)
2. \(\text{SD} = \sqrt{0.9^2 (0.0237)^2} = 0.1213\)
EXERCISE 4:

(a) \[ x_1 = \frac{166.2 - 22.4}{166.2 - 2 \times 22.4 + 220.4} = 0.42 \]

\[ x_2 = \frac{-22.4 + 220.4}{166.2 - 2 \times 22.4 + 220.4} = 0.58 \]

\(\text{or} \quad 1 - x_1 = 0.58\)

(b) \[ \hat{R}_b = x \hat{R}_a + (1 - x) R_f \]

\[ 0.0129724 = x (0.01315 \text{ S}) + (1 - x) 0.011 \]

\( x = 0.55 \) and \( 1 - x = 0.45 \)

55% in A and 45% in Rf

(c). YES, DRAW THE TANGENT LINE FROM Rf TO THE EFFICIENT FRONTIER TO FIND THE COMPOSITION OF PORTFOLIO G. THEN COMBINE G WITH Rf TO FIND B WHICH HAS HIGHER EXPECTED RETURN THAN B.