University of California, Los Angeles  
Department of Statistics  

Statistics C183/C283  
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Homework 5  

EXERCISE 1  
Assume the single index model holds and short sales are allowed. Use monthly data for the S&P500 index and for 5 stocks of your choice for the last five years. Choose a value of the riskless asset $R_f$. Verify using $R$ that the optimal portfolio (point of tangency $G$) is the same using two different methods. 

a. Finding directly the composition of point $G$ using $Z = \Sigma^{-1}R$.

b. Ranking the stocks based on the excess return to beta ratio.

EXERCISE 2  
Answer the following questions:

a. The betas of 30 stocks were obtained using simple regression in two successive periods: 2008-12-31 to 2013-01-31 (period 1) and 2013-02-28 to 2017-03-31 (period 2). There are 49 months in each period. Suppose we use the unadjusted betas in the first period as predictions of the betas in the second period. We can then compute the prediction sum of squares (PRESS) to evaluate the performance of these unadjusted betas. The following information is obtained from these data:

\[
\begin{align*}
\sum_{i=1}^{30} P_i &= 32.44349 & \text{Sum of the betas in period 1.} \\
\sum_{i=1}^{30} A_i &= 32.26206 & \text{Sum of the betas in period 2.} \\
\sum_{i=1}^{30} P_i^2 &= 51.70104 & \text{Sum of the squared betas in period 1.} \\
\sum_{i=1}^{30} A_i^2 &= 48.43207 & \text{Sum of the squared betas in period 2.} \\
\sum_{i=1}^{30} (A_i - \bar{A})(\hat{A}_i - \bar{A}) &= 4.299189 & \hat{A}_i \text{ are the fitted values of the simple regression of } A \text{ on } P.
\end{align*}
\]


b. An investor has $900000 invested in a diversified portfolio. Subsequently the investor inherits ABC company stock worth $100000. His financial adviser provided him with the following forecast information:

<table>
<thead>
<tr>
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<th>$R$ (monthly)</th>
<th>$\sigma$ (monthly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio</td>
<td>0.67%</td>
<td>2.37%</td>
</tr>
<tr>
<td>ABC Company</td>
<td>1.25</td>
<td>2.95</td>
</tr>
</tbody>
</table>

The correlation coefficient between ABC company stock returns and the portfolio is 0.40. Assume that the investor keeps the ABC company stock. Answer the following questions:

1. Calculate the expected return of the new portfolio which includes the ABC company stock.
2. Calculate the covariance between ABC company stock and the portfolio.
3. Calculate the standard deviation of his new portfolio which includes the ABC company stock.

c. Refer to question (b). If the investor sells the ABC company stock, he will invest the proceeds in risk-free government securities yielding 0.42% per month. Calculate the:

1. Expected return of the new portfolio which includes the government securities.
2. The standard deviation of his new portfolio which includes the government securities.