

**Homework 5**

**Exercise 1**

You are given the following data on 7 stocks:

Stock $i$	$\bar{R}_i$	$\sigma_i$
1	0.15	0.10
2	0.20	0.15
3	0.18	0.20
4	0.12	0.10
5	0.10	0.05
6	0.14	0.10
7	0.16	0.20

Using these data we ranked the stocks based on the excess return to standard deviation ratio ( $\frac{\bar{R}_i - R_f}{\sigma_i}$ ) to compute the entries in the next table. Assume  $R_f = 5\%$  and that the average correlation coefficient is  $\rho = 0.50$ .

Stock $i$	$\frac{\bar{R}_i - R_f}{\sigma_i}$	$\frac{\rho}{1 - \rho + i\rho}$	$\sum_{j=1}^i \frac{\bar{R}_j - R_f}{\sigma_j}$	$C_i$
1	1.00	0.5000	1.00	0.5000
2	1.00	0.3333	2.00	0.6667
5	1.00	0.2500	3.00	0.7500
6	0.90	0.2000	3.90	???
4	0.70	0.1667	4.60	0.7668
3	0.65	0.1429	5.25	0.7502
7	0.55	0.1250	5.80	0.7250

- Find the missing value  $C_4$ .
- What is the composition of the optimum portfolio assuming no short sales?
- What is the expected return and standard deviation of the combination of the optimum portfolio with the risk free asset (80% and 20%)? Show this combination on the graph of expected return against standard deviation.

**Exercise 2**

Assume that  $\sigma_m^2 = 10$ ,  $R_f = 0.05$ . You are also given  $\beta_1 = 1, \beta_2 = 1.5, \beta_3 = 1, \beta_4 = 2, \beta_5 = 1, \beta_6 = 1.5, \beta_7 = 2, \beta_8 = 0.8, \beta_9 = 1, \beta_{10} = 0.6$ . The table below shows the procedure for finding the cut-off point  $C^*$ .

Stock $i$	$\frac{\bar{R}_i - R_f}{\beta_i}$	$\frac{(\bar{R}_i - R_f)\beta_i}{\sigma_{\varepsilon_i}^2}$	$\sum_{j=1}^i \frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{\varepsilon_j}^2}$	$\frac{\beta_i^2}{\sigma_{\varepsilon_i}^2}$	$\sum_{j=1}^i \frac{\beta_j^2}{\sigma_{\varepsilon_j}^2}$	$C_i$
1	10.0	0.20	(a)	0.02000	0.02000	1.67
2	8.0	0.45	0.65	0.05625	0.07625	3.69
3	7.0	0.35	1.00	0.05000	0.12625	4.42
4	6.0	2.40	3.40	0.40000	0.52625	5.43
5	6.0	0.15	3.55	0.02500	0.55125	(c)
6	4.0	0.30	3.85	0.07500	0.62625	5.30
7	3.0	0.30	4.15	0.10000	(b)	5.02
8	2.5	0.10	4.25	0.04000	0.76625	4.91
9	2.0	0.10	4.35	0.05000	0.81625	4.75
10	1.0	0.06	4.41	0.06000	0.87625	4.52

- Find the three missing values, (a), (b), (c) in the table above.
- If short sales are not allowed find the cut-off point  $C^*$  and the value of  $z_1$ .
- If short sales are allowed find the cut-off point  $C^*$  and the value of  $z_1$ .
- Find the correlation coefficient between stock 1 and the market.

### Exercise 3

Using the constant correlation model we completed the table below on 12 stocks. Assume  $R_f = 0.05$  and average correlation  $\rho = 0.45$ .

Stock $i$	$\bar{R}_i$	$\sigma_i$	$\frac{R_i - R_f}{\sigma_i}$	$\frac{\rho}{1 - \rho + i\rho}$	$\sum_{j=1}^i \frac{R_j - R_f}{\sigma_j}$	$C_i$
1	0.27	0.031	7.097	0.450	7.097	3.194
2	0.31	0.042	6.190	0.310	13.287	4.124
3	0.16	0.023	4.783	0.237	18.070	4.280
4	0.15	0.021	4.762	0.191	22.832	4.372
5	0.33	0.059	4.746	$a = ?$	$b = ?$	$c = ?$
6	0.27	0.061	3.607	0.138	31.184	4.318
7	0.19	0.039	3.590	0.122	34.774	4.229
8	0.13	0.029	2.759	0.108	37.532	4.070
9	0.16	0.051	2.157	0.098	39.689	3.883
10	0.12	0.038	1.842	0.089	41.531	3.701
11	0.08	0.022	1.364	70.082	42.895	3.510
12	0.06	0.028	0.357	0.076	43.252	3.271

- Find the three missing numbers  $a, b, c$  in the table above.
- Find the cut-off point  $C^*$  if short sales are not allowed.
- Find the cut-off point  $C^*$  if short sales are allowed.
- Write down the expression in matrix form that computes the variance of the portfolio when short sales are allowed. No calculations.
- You are given a new stock with  $\bar{R} = 0.055, \sigma = 0.025$ . What will change when short sales are allowed and when short sales are not allowed in terms of the portfolio allocation. Briefly explain your answer without doing all the calculations.

### Exercise 4

Using the constant correlation model you constructed the following table.

	Rbar	Rbar_f	sigma	Ratio	col1	col2	col3
R8	0.0183190300	0.0180190300	0.14296181	0.126040859	0.21005601	0.1260409	0.02647564
R7	0.0076936650	0.0073936650	0.11188414	0.066083226	0.17359197	0.1921241	0.03335120
R3	0.0066229478	0.0063229478	0.12764631	0.049534905	0.14791510	0.2416590	0.03574501
R5	0.0045679107	0.0042679107	0.09613761	0.044393767	0.12885543	0.2860528	0.03685945
R9	0.0049333680	0.0046333680	0.10946825	0.042326134	0.11414698	0.3283789	0.03748346
R2	0.0035831364	0.0032831364	0.09725638	0.033757541	0.10245235	0.3621364	0.03710173
R1	0.0027625075	0.0024625075	0.07617847	0.032325505	0.09293132	0.3944619	0.03665787
R6	0.0028820487	0.0025820487	0.10005448	0.025806428	0.08502942	0.4202684	0.03573518
R4	0.0004543726	0.0001543726	0.09795510	0.001575953	0.07836601	0.4218443	0.03305825
R10	-0.0123139765	-0.0126139765	0.22000975	-0.057333716	0.07267106	0.3645106	0.02648937

Find  $z_1$ . Which stocks will be held long?

### Exercise 5

Answer the following questions:

- Consider the multi group model with 5 stocks in each of 5 industries. The solution is based on the calculation of the  $\Phi_i$  values which are computed using  $\Phi = A^{-1}C$ . Write the elements of the third column of matrix  $A$ . Write the expression for  $z_{13}$  (the  $z$  value for stock 13 that belongs in the third industry).
- Consider the multi-index model with 5 stocks in each of 5 industries. The solution is based on the calculation of the  $\Phi_i$  values which are computed using  $\Phi = M^{-1}R$ . Write the elements of the second row of matrix  $M$ . Write the expression for  $z_4$  (the  $z$  value for stock 4 that belongs in the second industry).

### Exercise 6

Answer the following questions:

- Suppose the single index model holds, short sales are allowed, and that the investor has access to the risk free asset. Show that the cut-off point  $C^*$  can be written as  $C^* = (\bar{R}_p - R_f)\beta_p \frac{\sigma_m^2}{\sigma_p^2}$ . Use your project data and your answer to project 5 (b,c) to support this result.
- You are given the following data:

Stock $i$	$\bar{R}_i$	$\sigma_i$
1	0.29	0.03
2	0.19	0.02
3	0.08	0.15

- Assume short sales are allowed,  $R_f = 0.05$ , and  $\bar{\rho} = 0.5$ . Rank the stocks based on the excess return to standard deviation ratio, find the cut-off rate  $C^*$ , and the optimal portfolio. Would the answer change if short sales are not allowed?
- The solution when short sales allowed could have been found using the techniques that discussed earlier in class through the following:

$$\mathbf{Z} = \Sigma^{-1} \mathbf{R} = \begin{pmatrix} 0.00090 & 0.00030 & 0.00225 \\ 0.00030 & 0.00040 & 0.00150 \\ 0.00225 & 0.00150 & 0.02250 \end{pmatrix}^{-1} \begin{pmatrix} 0.29 - 0.05 \\ 0.19 - 0.05 \\ 0.08 - 0.05 \end{pmatrix} = \begin{pmatrix} 280.00 \\ 320.00 \\ -48.00 \end{pmatrix}.$$

Explain what you see here and verify that the solution is the same as with part (1) when short sales are allowed.

### Exercise 7

For three stocks you are given the following data based on the single index model:

Stock	$\bar{R}$	$\beta$	$\sigma_\epsilon^2$
A	0.006	0.94	0.0033
B	0.011	0.61	0.0038
C	0.015	1.12	0.0046

- Rank the three stocks based on the excess return to beta ratio and complete the table below that will allow you to find the  $C^*$ . Assume that  $R_f = 0.005$ . and  $\sigma_m^2 = 0.0018$ .

Stock $i$	$\hat{\beta}_i$	$\bar{R}_i$	$\frac{\sigma_\epsilon^2}{\epsilon_i^2}$	$\frac{R_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\frac{\sigma_\epsilon^2}{\epsilon_i^2}}$	$\sum_{j=1}^i \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\frac{\sigma_\epsilon^2}{\epsilon_j^2}}$	$\frac{\hat{\beta}_i^2}{\frac{\sigma_\epsilon^2}{\epsilon_i^2}}$	$\sum_{j=1}^i \frac{\hat{\beta}_j^2}{\frac{\sigma_\epsilon^2}{\epsilon_j^2}}$	$C_i$

- Assume short sales are allowed. Find  $C^*$  and use it to find the composition of the optimal portfolio (point of tangency).
- Assume short sales are not allowed. Find  $C^*$  and use it to find the composition of the optimal portfolio.

### Exercise 8

Consider the multigroup model. In class we used an example of two stocks and two industries and then we extended the result to the general case. Use the same steps to develop the system of equations when there are three stocks and three industries to find the elements of the equation  $\mathbf{A}\Phi = \mathbf{C}$ . You can begin with

$$\bar{R}_1 - R_f = z_1\sigma_1^2 + z_2\sigma_{12} + z_3\sigma_{13} + z_4\sigma_{14} + z_5\sigma_{15} + z_6\sigma_{16} + z_7\sigma_{17} + z_8\sigma_{18} + z_9\sigma_{19}.$$