Homework 6 solutions

(a) \( \overline{R} \)

**Selectivity:**

\[
\overline{R}_A' = \overline{R}_F + \frac{\overline{R}_M - \overline{R}_F}{1} \cdot 0.5 = 0.0175
\]

\[
\therefore \overline{R}_A - \overline{R}_A' = 0.06 - 0.0175 = 0.0425
\]

**Net Selectivity:**

\[
0.0375 = \overline{R}_A' \cdot \sigma_m \Rightarrow \overline{R}_A'' = 1.5
\]

\[
\overline{R}_A'' = 0.001 + \frac{0.034 - 0.001}{1} \cdot 1.5 = 0.0505
\]

\[
\therefore \overline{R}_A - \overline{R}_A'' = 0.06 - 0.0505 = 0.0095
\]

**Densification:**

\[
\overline{R}_A'' - \overline{R}_A = 0.0505 - 0.0175 = 0.033
\]
\[
\bar{R}_T = R_F + \frac{\bar{E}_T - \bar{R}_F}{a_T}
\]
\[
= 0.001 + \left(\frac{0.034 - 0.001}{0.25}\right) = 0.00925
\]
\[
\therefore \bar{R}_T - R_F = 0.00825
\]
\[
\bar{R}_{T'} - \bar{R}_T = \left(0.001 + \left(0.034 - 0.001\right)0.5\right) - 0.00925
\]
\[
= 0.00825.
\]
A large pension fund wants to evaluate the performance of four portfolio managers for the last 5 years. During this time period the average annual return of the S&P500 was 14% with standard deviation 12%. The average annual risk free interest rate was 8%. The four portfolios gave the following data:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Average annual return (%)</th>
<th>Standard deviation (%)</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16</td>
<td>19</td>
<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>22</td>
<td>16</td>
<td>1.9</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>10</td>
<td>0.8</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>13</td>
<td>1.3</td>
</tr>
</tbody>
</table>

For funds A and B, how much the return on B has to change to reverse the ranking using the Sharpe measure?

We want \( \frac{\overline{R_B} - 8}{16} \leq \frac{16 - 8}{19} \)

\( \overline{R_B} \leq 8 + \frac{16 \times 8}{19} \) OR \( \overline{R_B} \leq 14.74 \)

(should drop from 22 to 14.74 (or less))
**Long Stress**

- $s_1 \leq 50$  
  - $s_1$
- $50 < s_1 \leq 60$  
  - $s_1$
- $s_1 > 60$  
  - $s_1$

**Short Stress**

- $E = 50$
  - $0$
- $E = G_1$
  - $0$
  - $s_1$
- $110 - s_1$
  - $0$
\[ P + S_0 = C + \frac{e}{1+r} \]

\[ S + \frac{105}{1.05} = 17 + \frac{105}{1.05} \]

\[ \text{Strategy: Sell Call} \]

\( \text{(Borrow 115 to buy } P + S_0) \)

\[ \frac{115}{1.05} = 117 \]

\[ \frac{-115}{0.98} = -117 \]

\( \text{Need to return 102.9} \)

\( S_1 > 105 \) \( \text{Sell at 105} \)

\( S_1 < 105 \) \( \text{Sell at 115} \)

\( 105 - 102.9 = 2.1 \text{ profit} \)
The price of a European put option on stock A is $4.0. The current price of the stock is $46, the exercise price of the put option is $E = 51$, time to expiration is 1 month, and the risk-free interest rate for the one-month period is 0.005. Is there an opportunity for riskless profit? If there is, please explain the positions you need to hold with the corresponding payoffs.

Lower bound: \( P > \frac{E}{1+i} - S_0 = \frac{51}{1.005} - 46 = 4.75 \)

But \( P = 4.0 \) (charade!)

Buy put at $4.0. Borrow 50 now (must return $50.25)

Buy stock at $46.0

At expiration:
If $S_1 > 51$, sell at $E = 51 \rightarrow 51 - 50.25 = +0.75$
If $S_1 < 51$, say $51 \rightarrow 51 - 50.25 = +0.75$
(Even better!)

One of the investing strategies involving options is the "protective put," where the investor buys the put and buys the stock. What position in call options is equivalent to this strategy? Please explain.

\[ P + S_0 = C + \frac{E}{1+i} \]

Buy put & is equivalent to buying call
Buy stock

The general payoff should be similar to the payoff of a call.

\[ \text{Payoff} \]

\[ E \]
Treznov for portfolio A:

\[
\frac{R_a - R_f}{\sigma_a} = \frac{0.16 - 0.08}{0.12} = 0.067
\]

Sharpe for portfolio B:

\[
\frac{R_s - R_f}{\sigma_s} = \frac{0.12 - 0.08}{0.16} = 0.25
\]

\[(h)\]

\[
\bar{R}_e = R_f + (\bar{R}_m - R_f) \beta_p
\]

A: \( \bar{R}_e = 0.08 + (0.14 - 0.08) 1.2 = 0.158 \)

B: \( \bar{R}_e = 0.08 + (0.14 - 0.08) 1.9 = 0.194 \)

C: \( \bar{R}_e = 0.08 + (0.14 - 0.08) 0.8 = 0.128 \)

D: \( \bar{R}_e = 0.08 + (0.14 - 0.08) 1.3 = 0.158 \)
Consider the multi-index model as discussed in class. Derived the covariance between stocks that belong in the same industry and the covariance between stocks that belong in different industries. Please show the details.

\[ \sigma_{ix} = \text{Cov} \left[ \alpha_i + b_i I_j + \varepsilon_i, \ \alpha_k + b_k I_j + \varepsilon_k \right] = b_i b_k \sigma_j = b_i b_k \left[ \text{Var}(\varepsilon_j) + \sigma_j^2 \right] \]

\[ \sigma_{ix} = \text{Cov} \left[ \alpha_i + b_i I_j + \varepsilon_i, \ \alpha_k + b_k I_1 + \varepsilon_k \right] = b_i b_k \text{Cov}(I_j, I_1) = b_i b_k \sigma_j \sigma_m \]
Consider the following two measures of portfolio performance: The Sharpe ratio and the differential excess return. Show graphically a situation of two portfolios $A$ and $B$ that are ranked as $A > B$ using the Sharpe ratio but at the same time $B > A$ using the differential excess return. Please explain why $A > B$ and $B > A$ for the respective measures of performance mentioned above.