Homework 7

Exercise 1
An investor sells a European call on a share for $4. The stock price is $47 and the exercise price is $50. When does the investor make a profit? When will the option be exercised? Draw a diagram showing the investor’s profit against the price of the stock at expiration.

Exercise 2
An investor buys a European put on a share for $3. The stock price is $42 and the exercise price is $40. When does the investor make a profit? When will the option be exercised? Draw a diagram showing the investor’s profit against the price of the stock at expiration.

Exercise 3
You want to purchase 2 puts and 1 call. The call option costs $5 and the put option costs $6. The exercise price for the call or the put is $50. Plot the profit against the stock price at the expiration date:
   a. For the 2 puts.
   b. For the call.
   c. For the combination of the 2 puts and 1 call.

Exercise 4
Consider the following strategy: You write 2 call options (each one with $E = 45, C = 5$) and you buy 1 call option (with $E = 40, C = 8$). Both buying and selling call options have the same expiration date. Plot the profit against the stock price at the expiration date for this strategy.

Exercise 5
Consider the box spread strategy: It is a combination of a bull call spread and a bear put spread.
   Bull call spread: Buy one call with exercise $E_1 = 50$ and sell one call with exercise $E_2 = 60$.
   Bear put spread: Buy one put with exercise $E_2 = 60$ and sell one put with exercise $E_1 = 50$.
   a. Complete the table that shows the payoffs for all the positions above.
   b. Construct the diagram that shows the payoff for the bull call spread, for the bear put spread, and the total (box spread).

Exercise 6
You want to find the value of a European call option for the following data: $S_0 = 50, E = 60, u = 1.2, d = \frac{1}{u}, r = 0.10$ (for each period), and number of periods to expiration $n = 10$. Using the binomial option pricing model:
   a. Find the value of $k$, the number of up movements of the stock, so that the call is “in the money” at the end of the $10_{th}$ period.
   b. Draw the binomial tree diagram and place the price of the stock at each node of the binomial tree (only at the end of $10_{th}$ period).
   c. What is the intrinsic value of the call at each node of the $10_{th}$ period?
   d. Find the price of the call at $t = 0$ by:
      1. Using the binomial formula:
         \[ C = S_0 \sum_{j=k}^{n} \binom{n}{j} p^j (1-p')^{n-j} - \frac{E}{(1+r)^n} \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j} \]
      2. Discounting the expected value of the call at the end of the $10_{th}$ period.
      Note: (1) and (2) must give the same answer.
Exercise 7
A stock price is currently $50. Over each of the next two 3-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per year with continuous compounding. What is the value of a 6-month European call option with strike price of $51? Complete the entire binomial tree diagram for the 2 periods. Place the price of the stock and the price of the call at each node on the binomial tree diagram.

Exercise 8
Refer to exercise 7. What is the value of a 6-month European put option with strike price of $51? Complete the entire binomial tree diagram for the 2 periods. Place the price of the stock and the price of the put at each node on the tree diagram. Verify that the European call and the European put prices satisfy the put-call parity.