

University of California, Los Angeles
Department of Statistics

Statistics C183/C283

Instructor: Nicolas Christou

Homework 8 - Solutions

Exercise 2

It is given: $u = 1.06$ and $d = 0.95$. Also the risk-free interest rate is $r = 0.05$ with continuous compounding. We need to find p

$$p = \frac{e^{rt} - d}{u - d} = \frac{e^{0.05 \frac{3}{12}} - 0.95}{1.06 - 0.95} = 0.5689, \text{ and therefore } 1 - p = 0.4311.$$

The price of the stock at the three terminal nodes are 56.18, 50.35, 45.125. Therefore the intrinsic value of the call will be for the top terminal node $c_{u^2} = 56.18 - 51 = 5.18$. For the other two terminal nodes the intrinsic value of the call is 0. The value of the option at $t = 0$ is calculated as follows:

$$c = \frac{5.18 \times 0.5689^2}{e^{0.05 \frac{6}{12}}} = 1.635.$$

Exercise 3

The data are the same as with exercise 7. The intrinsic value of the put at the three terminal nodes are 0, $51 - 50.35 = 0.65$, and $51 - 45.125 = 5.875$. Therefore the price of the put at $t = 0$ is calculated as follows:

$$p = \frac{0.65(2)(0.5689)(0.4311) + 5.875(0.4311)^2}{e^{0.05 \frac{6}{12}}} = 1.376$$

The put-call parity states that:

$$c + \frac{E}{e^{rt}} = p + s_0 \Rightarrow p = c + \frac{E}{e^{rt}} - s_0.$$

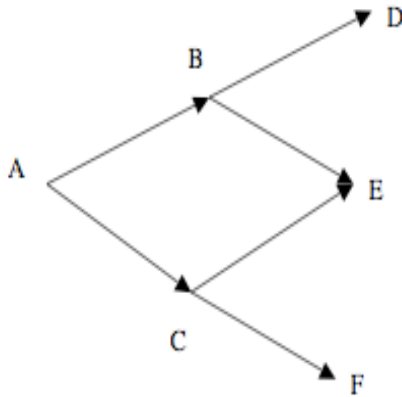
For our problem we have:

$$p = 1.635 + \frac{51}{e^{0.05 \frac{6}{12}}} - 50 = 1.376,$$

which verifies that the put-call parity holds.

Exercise 4

To answer this question we need to calculate the value of the put at each node (present value of the future payoff) and compare it to the payoff from early exercise. The put value at any node will be the greater between these two values. From the diagram below we observe that at node C the payoff from immediate exercise is $51 - 47.5 = 3.5$ which is larger than the value of the put (present value of payoff is 2.866). Therefore, if the put were American it would be optimal to exercise at node C .



The value of the put at node B is: $\frac{0.65 \times 0.4311}{e^{0.05 \frac{3}{12}}} = 0.2767$.

The value of the put at node C is 3.5.

Finally the value of the put at node A is: $\frac{0.2767 \times 0.5689 + 3.5 \times 0.4311}{e^{0.05 \frac{3}{12}}} = 1.646$.

Exercise 5

You are given: $S_0 = \$50$, $E = \$60$, $u = 1.2$, $d = \frac{1}{u} = 0.83333$, $r = 0.10$.

a. $u^k d^{10-k} 50 > 60 \Rightarrow k = 6$.

b. At the end of the 10_{th} period there are 11 terminal nodes on the binomial tree. The price of the stock at each terminal node is computed using $u^j d^{10-j} S_0$, $j = 0, \dots, 10$ and the value of the call is equal to $\max[u^j d^{10-j} S_0, 0]$.

Stock price	Call value
$(50)1.2^{10}0.83333^0 = 309.587$	249.587
$(50)1.2^9 0.83333^1 = 214.991$	154.991
$(50)1.2^8 0.83333^2 = 149.299$	89.299
$(50)1.2^7 0.83333^3 = 103.680$	43.680
$(50)1.2^6 0.83333^4 = 72.000$	12
$(50)1.2^5 0.83333^5 = 50.000$	0
$(50)1.2^4 0.83333^6 = 34.722$	0
$(50)1.2^3 0.83333^7 = 24.113$	0
$(50)1.2^2 0.83333^8 = 16.745$	0
$(50)1.2^1 0.83333^9 = 11.628$	0
$(50)1.2^0 0.83333^{10} = 8.075$	0

c. See table above.

d. Price of the call at $t = 0$:

1. Binomial formula: We need p and p' . $p = \frac{1+r-d}{u-d} = \frac{1+0.10-0.83333}{1.2-0.83333} = 0.72727$, and $p' = \frac{pu}{1+r} = \frac{0.72727 \times 1.2}{1+0.10} = 0.79339$.

$$C = 50 \sum_{j=6}^{10} \binom{10}{j} p'^j (1-p')^{10-j} - \frac{60}{(1+0.10)^{10}} \sum_{j=6}^{10} \binom{10}{j} p^j (1-p)^{10-j} = 27.486.$$

2. Discounting the expected value of the call at the end of the 10_{th} period:

$$\begin{aligned} C &= \frac{p^{10} 249.587 + \binom{10}{9} p^9 (1-p)^1 154.991 + \binom{10}{8} p^8 (1-p)^2 89.299}{(1+0.10)^{10}} \\ &+ \frac{\binom{10}{7} p^7 (1-p)^3 43.680 + \binom{10}{6} p^6 (1-p)^4 12}{(1+0.10)^{10}} = 27.486. \end{aligned}$$