

Constructing the optimal portfolios - Single index model
Calculation steps

- a. **Step 1:** By regressing the returns of each stock on the returns of the market obtain for each stock: $\hat{\beta}_i, \hat{\alpha}_i, \hat{\sigma}_{\epsilon_i}^2$ and construct the table below:

Stock i	$\hat{\alpha}_i$	$\hat{\beta}_i$	\bar{R}_i	$\hat{\sigma}_{\epsilon_i}^2$	$\frac{R_i - R_f}{\hat{\beta}_i}$
IBM					
GOOGLE					
⋮					

- b. **Step 2:** Sort the table above based on the excess return to beta ratio:

$$\frac{\bar{R}_i - R_f}{\beta_i}$$

- c. **Step 3:** Create 5 columns to the right of the sorted table as follows:

Stock i	$\hat{\alpha}_i$	$\hat{\beta}_i$	\bar{R}_i	$\hat{\sigma}_{\epsilon_i}^2$	$\frac{R_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\hat{\sigma}_{\epsilon_i}^2}$	$\sum_{j=1}^i \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\hat{\sigma}_{\epsilon_j}^2}$	$\frac{\hat{\beta}_i^2}{\hat{\sigma}_{\epsilon_i}^2}$	$\sum_{j=1}^i \frac{\hat{\beta}_j^2}{\hat{\sigma}_{\epsilon_j}^2}$	C_i
					k_1	k_1	k_1	l_1	l_1	C_1
					k_2	$k_1 + k_2$	$k_1 + k_2$	l_2	$l_1 + l_2$	C_2
					k_3	$k_1 + k_2 + k_3$	$k_1 + k_2 + k_3$	l_3	$l_1 + l_2 + l_3$	C_3
					⋮	⋮	⋮	⋮	⋮	⋮
					k_n	$k_1 + k_2 + \dots + k_n$	$k_1 + k_2 + \dots + k_n$	l_n	$l_1 + l_2 + \dots + l_n$	C_n

Note: Compute all the $C_i, i = 1, \dots, n$ (last column) as follows:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^i \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\hat{\sigma}_{\epsilon_j}^2}}{1 + \sigma_m^2 \sum_{j=1}^i \frac{\hat{\beta}_j^2}{\hat{\sigma}_{\epsilon_j}^2}} = \frac{\sigma_m^2 \times \text{COL2}}{1 + \sigma_m^2 \times \text{COL4}}$$

Once the C'_i 's are calculated we find the C^* as follows:

If short sales are allowed, C^* is the last element in the last column.

If short sales are not allowed, C^* is the element in the last column for which $\frac{\bar{R}_i - R_f}{\beta_i} > C^*$.

In both cases the z'_i 's are computed as follows

$$z_i = \frac{\beta_i}{\sigma_{\epsilon_i}^2} \left(\frac{\bar{R}_i - R_f}{\beta_i} - C^* \right)$$

and the x'_i 's

$$x_i = \frac{z_i}{\sum_{i=1}^n z_i}$$