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Statistics C183/C283

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**The efficient frontier with short sales allowed**

An investor can often sell a security that he does not own. This process is called “short selling”. So far we have assumed that short sales are not allowed which means  $x_i \geq 0$ ,  $i = 1, \dots, n$ . When short sales are allowed some of the  $x_i$  can be negative but as before always  $\sum_{i=1}^n x_i = 1$ . With short sales allowed the investor can achieved larger expected returns but as we will see below this large increase in the expected return is associated with a very large increase in risk.

Consider the following example with 2 stocks.

	$A$	$B$
$\bar{R}$	0.14	0.08
$\sigma$	0.06	0.03

In addition  $\rho_{AB} = 0.5$ .

If short sales are not allowed the largest expected return the investor can have is 14% (invest all his wealth in stock  $A$ ). However, with short sales allowed higher expected returns can be achieved by short selling stock  $B$  (borrow  $B$  and sell it). The proceeds are used to buy more shares of stock  $A$ . For example, suppose the investor initially has \$100 to invest. The investor can short \$1000 worth of stock  $B$  and invest now \$1100 in stock  $A$ .

Therefore,  $x_A = 11$  and  $x_B = -10$ . The expected return and standard deviation of this combination are:

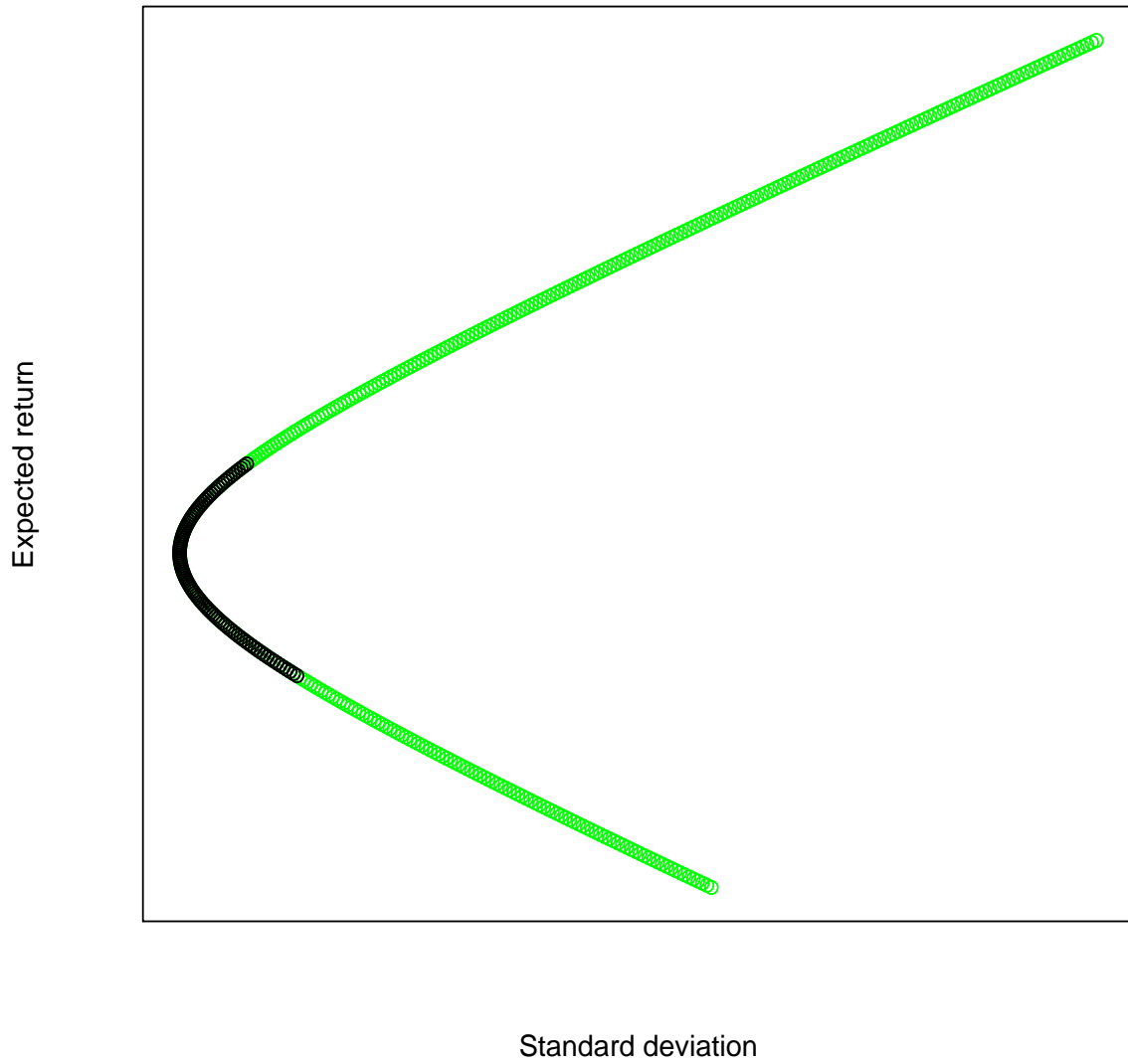
$$\bar{R}_p = x_A \bar{R}_A + x_B \bar{R}_B = 11(0.14) - 10(0.08) = 0.74.$$

and

$$\sigma_p = \sqrt{x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \rho_{AB} \sigma_A \sigma_B} \Rightarrow$$
$$\sigma_p = \sqrt{11^2(0.06)^2 + 10^2(0.03)^2 + 2(11)(-10)0.5(0.06)(0.03)} = 0.572$$

The shape of the efficient frontier is very similar to the case when short sales are not allowed, however there is no finite upper bound. In the case of shorts sales not allowed the efficient frontier starts with the minimum risk portfolio and ends with the maximum expected return portfolio. The next figure shows the portfolio possibilities curve with and without short sales.

# Portfolio possibilities curve with and without short sales



# Short sales allowed - example

rho=0.11704

	x1	x2	var(return)	sd(return)	E(return)
	1.00	0.00	0.0061	0.078102	0.01
	0.95	0.05	0.005576	0.07467	0.01015
	0.90	0.10	0.005099	0.071404	0.0103
	0.85	0.15	0.004669	0.068329	0.01045
	0.80	0.20	0.004286	0.065471	0.0106
	0.75	0.25	0.003951	0.062859	0.01075
	0.70	0.30	0.003663	0.060526	0.0109
	0.65	0.35	0.003423	0.058505	0.01105
Rbar	0.01	0.013	0.00323	0.05683	0.0112
Var	0.0061	0.0046	0.003084	0.055531	0.01135
	0.50	0.50	0.002985	0.054635	0.0115
Cov	0.00062		0.002934	0.054163	0.01165
	0.45	0.55	0.002934	0.054163	0.01165
	0.40	0.60	0.00293	0.054126	0.0118
	0.35	0.65	0.002973	0.054524	0.01195
	0.30	0.70	0.003063	0.055348	0.0121
	0.25	0.75	0.003201	0.05658	0.01225
	0.20	0.80	0.003386	0.058193	0.0124
	0.15	0.85	0.003619	0.060157	0.01255
	0.10	0.90	0.003899	0.062439	0.0127
	0.05	0.95	0.004226	0.065005	0.01285
	0.00	1.00	0.0046	0.067823	0.013
	1.05	-0.05	0.006672	0.08168	0.00985
	1.1	-0.1	0.007291	0.085385	0.0097
	1.15	-0.15	0.007957	0.089201	0.00955
	1.2	-0.2	0.00867	0.093115	0.0094
	1.25	-0.25	0.009431	0.097115	0.00925
	1.3	-0.3	0.010239	0.10119	0.0091
	1.35	-0.35	0.011095	0.105332	0.00895
	1.4	-0.4	0.011998	0.109534	0.0088
	1.45	-0.45	0.012948	0.113788	0.00865
	1.5	-0.5	0.013945	0.118089	0.0085
	1.55	-0.55	0.01499	0.122432	0.00835
	1.6	-0.6	0.016082	0.126813	0.0082
	1.65	-0.65	0.017221	0.131228	0.00805
	1.7	-0.7	0.018407	0.135674	0.0079
	1.75	-0.75	0.019641	0.140147	0.00775
	1.8	-0.8	0.020922	0.144646	0.0076
	1.85	-0.85	0.022251	0.149167	0.00745
	1.9	-0.9	0.023627	0.153709	0.0073
	1.95	-0.95	0.02505	0.158271	0.00715
	2	-1	0.02652	0.16285	0.007
	-0.05	1.05	0.005022	0.070864	0.01315
	-0.1	1.1	0.005491	0.074099	0.0133
	-0.15	1.15	0.006007	0.077504	0.01345
	-0.2	1.2	0.00657	0.081058	0.0136
	-0.25	1.25	0.007181	0.084742	0.01375
	-0.3	1.3	0.007839	0.08854	0.0139
	-0.35	1.35	0.008545	0.092438	0.01405
	-0.4	1.4	0.009298	0.096424	0.0142
	-0.5	1.5	0.010945	0.104618	0.0145
	-0.45	1.45	0.010098	0.100487	0.01435
	-0.55	1.55	0.01184	0.10881	0.01465
	-0.6	1.6	0.012782	0.113056	0.0148
	-0.65	1.65	0.013771	0.117349	0.01495
	-0.7	1.7	0.014807	0.121686	0.0151
	-0.75	1.75	0.015891	0.126061	0.01525
	-0.8	1.8	0.017022	0.13047	0.0154
	-0.85	1.85	0.018201	0.134911	0.01555
	-0.9	1.9	0.019427	0.139379	0.0157
	-0.95	1.95	0.0207	0.143874	0.01585
	-1	2	0.02202	0.148391	0.016

