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Statistics C183/C283 **Instructor: Nicolas Christou**

Point of tangency - example

We will use an example with 5 stocks. The data can be accessed at

```
data <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/
        returns_5stocks.txt", header=TRUE)
```

Compute the mean returns and the variance-covariance matrix $\mathbf{\Sigma}$:

```
> R_ibar <- as.matrix(mean(a))
> R_ibar
          [,1]
R1 0.006362437
R2 0.002201256
R3 0.011741093
R4 0.010026242
R5 0.013801266

#Compute the variance-covariance matrix:
> var_covar <- cov(a)
> var_covar
            R1          R2          R3           R4           R5
R1 0.010231184 0.004570583 0.004178472 0.0012245825 0.0043699375
R2 0.004570583 0.012351573 0.002601227 0.0011180271 0.0022117252
R3 0.004178472 0.002601227 0.012070071 0.0014954732 0.0058545832
R4 0.001224582 0.001118027 0.001495473 0.0142320539 0.0005037049
R5 0.004369937 0.002211725 0.005854583 0.0005037049 0.0145066003
```

Compute the inverse of the variance-covariance matrix $\mathbf{\Sigma^{-1}}$:

```
> var_covar_inv <- solve(var_covar)
> var_covar_inv
           R1          R2         R3        R4          R5
R1 137.702357 -40.5528945 -26.449937 -5.018123 -24.4494104
R2 -40.552894  97.6584652  -6.636531 -3.489538   0.1262702
R3 -26.449937  -6.6365311 111.785787 -7.679539 -35.8683780
R4  -5.018123  -3.4895385  -7.679539 71.682853   2.6539836
R5 -24.449410   0.1262702 -35.868378  2.653984  90.6636067
```

Create the vector $\boldsymbol{R}$ (assume $R_f = 0.002$):

```
> Rf <- 0.002
> R <- R_ibar-Rf
> R
           [,1]
R1 0.0043624371
R2 0.0002012564
R3 0.0097410929
R4 0.0080262421
R5 0.0118012661
```

Compute the vector $\boldsymbol{Z} = \boldsymbol{\Sigma}^{-1}\boldsymbol{R}$:

```
> z <- var_covar_inv %*% R
> z
           [,1]
R1  0.006094381
R2 -0.248419860
R3  0.487263793
R4  0.509263661
R5  0.635216055
```

Compute the vector $\boldsymbol{X}$:

```
> x <- z/sum(z)
> x
           [,1]
R1  0.004386283
R2 -0.178794182
R3  0.350696322
R4  0.366530195
R5  0.457181382
```

Compute the expected return of portfolio G:

```
> R_Gbar <- t(x) %*% R_ibar
> R_Gbar
           [,1]
[1,] 0.01373650
```

Compute the variance and standard deviation of portfolio G:

```
> var_G <- t(x) %*% var_covar %*% x
> var_G
            [,1]
[1,] 0.008447059

> sd_G <- var_G^0.5
> sd_G
           [,1]
[1,] 0.09190788
```

Compute the slope of the tangent line to the efficient frontier:

```
> slope <- (R_Gbar-Rf)/(sd_G)
> slope
          [,1]
[1,] 0.1276985
```

We can now draw the line because we have two points (0, 0.002) and (sd_G, R_Gbar). Let's find one more point (borrowing segment):

```
(1.3*sd_G,  0.002+slope*(1.3*sd_G))
```

Before we draw the tangent, let's create many portfolios using different combinations of the five stocks:

```
return_p <- rep(0,10000000);
sd_p <- rep(0,10000000);
j <- 0
i <- 0

for (a in seq(-.2, 1, 0.1)) {
for (b in seq(-.2, 1, 0.1)) {
for(c in seq(-.2, 1, 0.1)){
for(d in seq(-.2, 1, 0.1)){
for(e in seq(-.2, 1, 0.1)){
if(a+b+c+d+e==1) {
j=j+1

return_p[j]=a*mean(data[,1])+b*mean(data[,2])+
c*mean(data[,3])+d*mean(data[,4])+e*mean(data[,5])

sd_p[j]=(a^2*var(data[,1]) +
b^2*var(data[,2])+
c^2*var(data[,3])+
d^2*var(data[,4])+
e^2*var(data[,5])+
2*a*b*cov(data[,1],data[,2])+
2*a*c*cov(data[,1],data[,3])+
2*a*d*cov(data[,1],data[,4])+
2*a*e*cov(data[,1],data[,5])+
2*b*c*cov(data[,2],data[,3])+
2*b*d*cov(data[,2],data[,4])+
2*b*e*cov(data[,2],data[,5])+
2*c*d*cov(data[,3],data[,4])+
2*c*e*cov(data[,3],data[,5])+
2*d*e*cov(data[,4],data[,5]))^.5
}
}
}
}
}
}

R_p <- return_p[1:j]
sigma_p <- sd_p[1:j]
```

We can get the same results if we use vectors and matrices as follows:

```
j <- 0
return_p <- rep(50000)
sd_p <- rep(0,50000)
vect_0 <- rep(0, 50000)
fractions <- matrix(vect_0, 10000,5)


for (a in seq(-.2, 1, 0.1)) {
for (b in seq(-.2, 1, 0.1)) {
for(c in seq(-.2, 1, 0.1)){
for(d in seq(-.2, 1, 0.1)){
for(e in seq(-.2, 1, 0.1)){
if(a+b+c+d+e==1) {
j=j+1
        fractions[j,] <- c(a,b,c,d,e)
         sd_p[j] <-  (t(fractions[j,]) %*% var_covar %*% fractions[j,])^.5
        return_p[j] <-  fractions[j,] %*% R_ibar

}
}
}
}
}
}
R_p <- return_p[1:j]
sigma_p <- sd_p[1:j]
```

On the expected return standard deviation space we can plot all these portfolios, place the five stocks, and draw the tangent line:

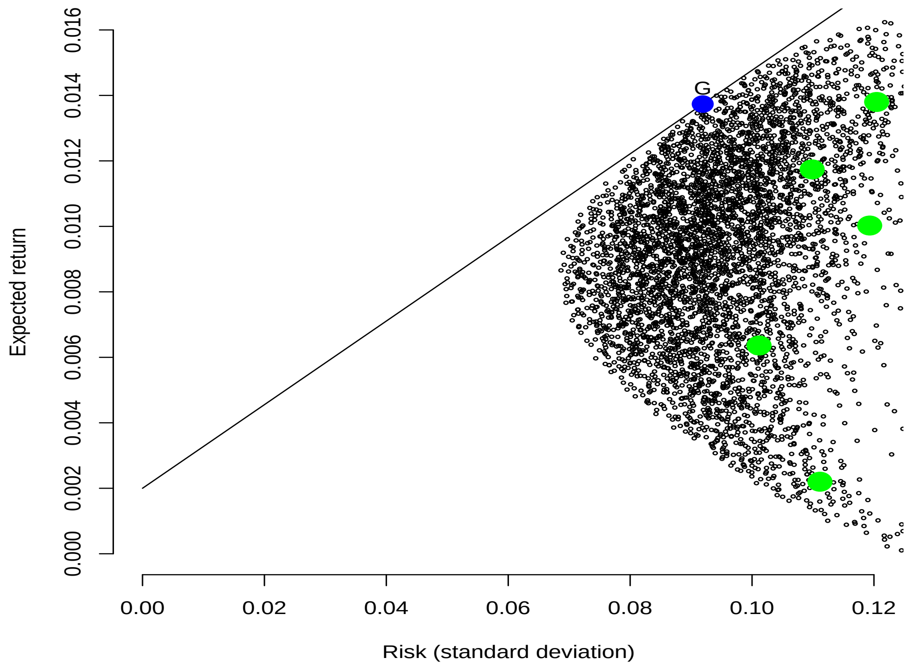
```
plot(sigma_p, R_p,xlab="Risk (standard deviation)", ylab="Expected return",
     xlim=c(0.0,.12), ylim=c(0.0,.016),axes=FALSE, cex=0.4)
axis(1, at=c(0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.12))
axis(2,at=c(0,0.002, 0.004, 0.006, 0.008, 0.010, 0.012, 0.014, 0.016))

lines(c(0,sd_G, 1.3*sd_G),c(.002,R_Gbar,0.002+slope*(1.3*sd_G)))

#Identify portfolio G:
points(sd_G, R_Gbar, cex=2, col="blue", pch=19)
text(sd_G, R_Gbar+.0005, "G")

#Plot the 5 stocks:
points(sd(data), mean(data), pch=19, cex=2.3, col="green")
```

The plot:



G
Expected return
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0.014
0.016
0.00
0.02
0.04
0.06
0.08
0.10
0.12
Risk (standard deviation)