Midterm 2, Math 154A, Lecture 1, Nov. 17, 1995 Name:

Section Number: ID:

Instructions No books or notes allowed. You may use a calculator. There are blank sheets at the end of the exam for your scratch work. To receive full credit, you **must** show your work. Simply writing the answer is not sufficient for full credit. Please circle your answers. You have fifty minutes.

1. Let X be a discrete random variable such that $\mathrm{P}(X = k) = \frac{k}{21}$ for $k = 1, 2, \ldots, 6$ and 0 for all other values of k. Let $Z = 3X - 2$.
   (a) (6 pts) Find $\mathrm{E}(X)$
   (b) (6 pts) Find $\mathrm{VAR}(X)$
   (c) (6 pts) Find $\mathrm{E}(Z)$
   (d) (7 pts) Find $\mathrm{VAR}(Z)$

2. Let $f_{xy}(x, y) = \frac{1}{18}xy$ for (x, y) in the region R, where $R = \{(x, y) : 0 \leq x \leq 3, 1 \leq y \leq 3\}$. Calculate:

   (a) (5 pts) $\mathrm{E}(XY)$

   (b) (5 pts) $\mathrm{E}(X)$

   (c) (5 pts) $\mathrm{E}(Y)$

   (d) (10 pts)$\mathrm{VAR}(X + Y)$

3. During a classical music concert during the flu season, there are approximately 2.5 coughs per minute from audience members. Assume that it is impossible for two people to start coughing at precisely the same moment.

   (a) (10 pts) Explain why the Poisson distribution is a suitable model for this problem.

   (b) (10 pts) A particular quiet passage in the piece being played lasts for one minutes. What is the probability that there will be no coughs during this passage? What is the probability that there are more than two coughs?

   (c) (5 pts) What is the expected number of coughs? The variance of the number of coughs?

4. Let $X_1, \ldots, X_n$ be independent random variables, each following a normal distribution with $\mathrm{E}(X_i) = \mu$ and $\mathrm{VAR}(x_i) = \sigma^2, i = 1, \ldots, n$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

   (a) (5 pts) Show that $\mathrm{E}(\bar{X}) = \mu$. Show each step and explain any assumptions you need to make it.

   (b) (8 pts) Find $\mathrm{VAR}(\bar{X})$. Show each step and justify any assumptions you need to make it.

   (c) Suppose $\mu = 10$, $\sigma^2 = 100$, and $n = 25$. Find

      i. (3 pts) $\mathrm{P}(7 \leq X \leq 13)$ and

      ii. (4 pts) $\mathrm{P}(7 \leq \bar{X} \leq 13)$.

   (d) (5 pts) The true weight of a certain rare metal is 1 kg. However, independent measurements of the weight have a certain amount of error, and therefore are normally distributed with mean 1 kg and variance .005 kg. Suppose you did not know the true weight. If it was very important for you to know the truth, which researcher would you trust the most: the one who takes a single observation, or the one who takes 5 observations and tells you the average. Why?