

Hw#8

1. a) $P(x < 11.85) = P(Z < (11.85 - 12) / 0.2) = P(Z < -0.75) = 0.7734$
 c) $12/128 = 0.09375$
 $0.2/128 = 0.0015625$
 d) $P(x_1 + x_2 > 24) = 1 - P(x_1 + x_2 < 24) = 1 - P(z < [(24 - 24) / (\sqrt{2} * 0.2)])$
 $= 1 - P(z < 0) = 0.5$
 $P(x_1 + x_2 > 25) = 1 - P(x_1 + x_2 < 25) = 1 - P(z < [(25 - 24) / (\sqrt{2} * 0.2)])$
 $= 1 - P(z < 3.536) = 0$
 e) $E(6x) = 6 * E(x) = 6 * 12 = 72$
 $SD(\text{total}) = \sqrt{6} * 0.2 = 0.49$
 f) $E(\text{average}) = 12$
 $SD(\text{average}) = 0.2 / \sqrt{6} = 0.0816$
 g) $P(\text{ave of 6 bottle} < 11.85) = P(Z < (12 - 11.85) / 0.0816)$
 $= P(Z < 1.838) = 0.9671$

2. a) pdf:

x	16	-1
p	1/19	18/19

- b) expected value = $16 * (1/19) + (-1) * (18/19) = -2/19$
 $SD = \sqrt{[(-1 + 2/19)^2 * (18/19) + (16 + 2/19)^2 * (1/19)]}$
 $= \sqrt{5202} / 19 \approx 3.8$
 c) expected value of $Y = 2 * E(X) = -4/19$
 $SD(Y) = \sqrt{2} * 3.8 = 5.37$
 d) pdf of Y

32 (win twice)	15(1 st win 16, 2 nd -1)	15(1 st -1, 2 nd win 16)	-2 (lost twice)
$(1/19)^2$	$(1/19) * (18/19)$	$(18/19) * (1/19)$	$(18/19)^2$

- e) $(1/19)^2 + (1/19) * (18/19) + (18/19) * (1/19) = 0.1025$
 or $1 - (18/19)^2 = 0.1025$

f) $E(\text{total}) = (-2/19) * 50 = -5.263$
 $SD(\text{total}) = \sqrt{50} * SD(X) = 26.842$
 $P(\text{total} > 0) = 1 - P(\text{total} < 0) = 1 - P[Z < (0 + 5.263) / 26.842] = 1 - P(Z < 0.196)$
 $= 1 - 0.5793 = 0.4207$

You can also use the pdf to do this question, using 1 minus the P of losing 50,49,48 times. when lose 47 times, the total win will larger than 0)

g)

Z	-50	-33	-16	1	18.....
P	$(18/19)^{50}$	$50 * (18/19)^{49} * (1/19)$	$(2 \text{ out } 50) * (18/19)^{48} * (1/19)^2$	$(3 \text{ out } 50) * (18/19)^{47} * (1/19)^3$

3. a) pdf of Y1

Y1	0	1
P	1/2	1/2

- b) $E(Y1) = 1/2$
 $SD(Y1) = \sqrt{(0-0.5)^2 * 0.5 + (1-0.5)^2 * 0.5} = 1/2$
- c) $E(X) = E(Y1 + Y2 + Y3 \dots) = N * (1/2)$
 $SD(X) = \sqrt{\text{Var}(X)} = \sqrt{N * \text{Var} Y1} = \sqrt{N} * SD(Y1) = \sqrt{N} * 0.5$
 You can also use $\sqrt{n * p * (1-p)} = \sqrt{n} * (1/2)$
- d) If thinking it as normal distribution,
 $P(X > 6) = 1 - P(X < 6) = 1 - P[Z < (6 - 10 * 0.5) / (\sqrt{10} * 0.5)] = 1 - P(Z < 0.63)$
 $= 1 - 0.7357 = 0.2643$ ($n * p < 10$, so it is not accurate)
- e) If toss 100 times, $n * p$ is large enough to use CLT
 $E = 100 * 0.5 = 50$ $SD = \sqrt{100} * 0.5 = 5$
 $P(55 < X < 65) = P[(55 - 50) / 5 < Z < (65 - 50) / 5] = P(1 < Z < 3)$
 $= P(z < 3) - P(z < 1) = 1 - 0.8413 = 0.1587$

4. a) fix number of trials ($n = 1000$);

Only two results: ate or not;
 the probability of each trial is the same ($= 0.45$);
 Each trial is independent;
 X represent the number out of 1000 who ate sushi last month;

b) $E = 1000 * 0.45 = 450$
 $SD = \sqrt{1000 * 0.45 * 0.55} = 15.73$

$$\begin{aligned} \text{c) } P(X > 500) &= 1 - P(X < 500) = 1 - P[Z < (500 - 450) / 15.73] \\ &= 1 - P(Z < 3.17) = 0 \end{aligned}$$

Section 7.1 P301

1. a) sample: 40 college freshmen
Population: all freshmen
 - b) sample: a few small pieces of rock
population: rock in river valley region
 - c) sample: twenty frogs
population: tree frogs in Minnesota.
 - d) sample: water sample from 30 locations in the Gulf
population: all water of the Gulf of Mexico
 - e) sample: five small samples
population: concrete of the building
 - f) sample: 20 blood samples
population: Lisa's blood pressure anytime
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5. a) clams in a particular river
 - b) the percentage of the clams which are safe
 - c) percentage of the sample that are safe

Section 7.2 P311

1. a) null hypothesis H_0 :
tendency of roll over of SUV = that of full-size car
Or $P(\text{SUV}) - P(\text{full size car}) = 0$
 - b) H_0 : public favor reducing the size of the military budget = $2/3$
 - c) H_0 : the prob of contract lung cancer of second hand smoke = that of people who have no exposure at all to cigarette smoke
 - d) H_0 : the probability of an accident in winter = that in summer
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2. a) the statistics is the difference $\hat{p}_1 - \hat{p}_2$ between SUV and full size car. The difference in these two samples would have to be improbably larger than zero under the null hypothesis for us to decided to reject the null hypothesis.
 - b) the observed \hat{P} is so much larger than $2/3$ that it is very unlikely to result from typical random variation around the $P = 2/3$ based on the assumption that the null hypothesis is true.

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- 3 a) the null hypothesis is $p=40\%$
- b) $p\text{-value}=0.2$ when assuming the null hypothesis is true.
 $0.2 > 0.05$ so fail to reject.

Section 7.3 P323

1. a) null hypothesis : $p(\text{agree})=1/2$
 - b) c) If one simulation consists of drawing a ball from the box (one labeled 1 to represent agree and one labeled 0 to represent not) 20 times with replacement, the total number of ones is recorded for each simulation. We know $20 \times 0.75 = 15$. Repeated the simulations and recorded how often the event of interest (number of ones ≥ 15) occurs. Or how often the $p \geq 0.75$.
 - d) most frequently occurring value of \hat{p} is 0.5
 least frequently occurring values: $\hat{p} = 0, 0.05, 0.1, 0.9, 0.95, 1$.
 least frequently occurring values in the direction of rejecting the null hypothesis is 0.9, 0.95, 1.
 - e) no
 - f) $(12+3+3)/1000 = 0.018 < 0.05$ so, reject the null hypothesis.
 - g) $(75+36+12+3+3)/1000 = 0.129 > 0.05$ then, fail to reject.
3. b) null hypothesis: Population proportion supporting such action = 50%
 - c) d) e) the box has two balls, one labeled 1 to represent supporting and one labeled 0 to represent not.
 one simulation is drawing a ball 160 times with replacement, the total number of ones is recorded for each simulation and calculate the p of supporting for each simulation.
 Repeated the simulations and recorded how often the p of supporting ≥ 0.51
 - f) $244/1000 = 0.244 > 0.05$ then fail to reject.