Finding $p$-Values can be a difficult part of this class because each professor and textbook covers it differently. When a textbook does not explain something clearly, it is a good idea to look in another book. However, in the case of $p$-values, using another book may confuse you even more.

Here are some reasons why computing $p$-values is different in many books:

- some books provide only one $z$ table (usually negative $z$ values)
- most books provide two tables, one for negative values and one for positive values.
- some books also add tables for two-tailed tests.
- Most books provide $p$-values for $P(Z < z)$.
- Few books provide tables for $P(Z > z)$.
- $p$-values for $t$ tests cannot be computed directly without a computer or calculator, even though the $t$ distribution is symmetric because $t$ distribution depends on degrees of freedom.
- $p$-values for $\chi^2$ and $F$ tests cannot be computed in the same way as with $z$ scores because there are degrees of freedom associated with the distributions, and the distributions are not even symmetric.

Definition of $p$-value:

The probability that a test statistic is at least as extreme as the one you computed, assuming that $H_0$ is true.

The definition may not mean much, but if you understand it, you will never get confused.

When performing a hypothesis test, it is important to understand where the rejection region is. When a test statistic falls in the rejection region, we reject the null hypothesis, otherwise we fail to reject $H_0$ in favor of $H_A$ or $H_1$.

$p$-values are always computed in the direction of the rejection region(s). Here, I provide an explanation for each type of alternative hypothesis.
One-Tailed $H_A$

A one-tailed alternate has a $>$ or $<$ in it. It is called one-sided, or *one-tailed* because there is only one rejection region.

$H_A : \mu < \mu_0$

The first case is the simplest because textbooks contain a $z$ table for a region rejection in the left tail of the distribution (referred to as $P(Z < z)$).

**Case 1: $z$ negative**

You’re in luck. Just look up the $z$ value in the table and report the $p$-value provided. If you have a positive and a negative $z$ table, obviously use the negative $z$ table.

**Case 2: $z$ positive**

Draw the distribution. You have something that looks like the following. You have a rejection region on the left side, and your $z$ score is somewhere on the right side. The $p$-value is computed *towards* the rejection region (hence the left arrow). Since $H_A$ is less-than, the $p$-value is the probability that a $z$ value is less than the $z$ value you computed.

If you are given both a negative and positive $z$ table, use the positive table and simply report the $p$-value given.

If you are given *just a negative* table, we have to be a bit creative. Note that the normal distribution is symmetric. This means that

$$P(Z > +z) = P(Z < -z)$$

Note that the shaded areas in the diagrams below are the same!

This means that if you only have a negative table and $H_A$ is $<$, and $z$ is positive, then look up the *negative* of the $z$ value and subtract from 1, because

$$1 - P(Z < -z) = P(Z > -z) = P(Z < +z)$$
\( H_A : \mu > \mu_0 \)

The > is a little more difficult because most books do not contain a table with an upper rejection region, but it is easy once we use the properties of the normal distributions.

**Case 1: \( z \) negative** The rejection region is on the right side of the distribution, and our \( z \) score is negative. Recall that we compute the \( p \) in the direction of the rejection region. We need to compute \( P(Z > z) \) where \( z \) is negative.

Using the negative \( z \) table, look up \( z \) and get a probability. If we subtract that probability from 1, we get the \( p \)-value.

\[
P(Z > -z) = 1 - P(Z < -z)
\]

**Case 2: \( z \) positive** The rejection region is still on the right side of distribution, but our \( z \) score is also on the right side (it is positive).

If we have a positive \( z \) table, we just look up the \( z \) score and subtract the probability from 1.

\[
P(Z > +z) = 1 - P(Z < +z)
\]

If we have only a negative \( z \) table, we look up the *negative* of the \( z \) score.

\[
P(Z < -z) = P(Z > +z)
\]

Regardless of these cases, for a one-tailed test, compare the \( p \)-value to \( \alpha \). If \( p \)-value < \( \alpha \), then we reject \( H_0 \).
**Two-Tailed $H_A$**

A two-tailed alternate has a $\neq$ in it. It is called two-sided, or *two-tailed* because there are two rejection regions, one on the left, and one on the right. Due to symmetry, the two tails always have equal area.

Computing $p$-values for two-tailed tests is actually easier! We have two critical values: one negative, and one positive. (The absolute values of the critical values are the same)

Look up the negative $z$ value to get a probability. Then what? Two choices:

1. compare it to $\frac{\alpha}{2}$ and if the probability is less than $\frac{\alpha}{2}$, then reject $H_0$.

\[
\begin{align*}
\text{ } & = 2 \times \\
\end{align*}
\]

2. multiply the probability by 2, and this can then be called the $p$-value. Compare the $p$-value to $\alpha$. If $p < \alpha$, reject $H_0$.

**IMPORTANT!**  Although I have used $\mu$ and $\mu_0$ throughout this handout, the procedure for computing the $p$-value for a $z$ test is the same for proportion tests. Use $p$ or $p_i - p_j$ instead of $\mu$ and $p_0$ or $p_i - p_{j0}$ instead of $\mu_0$. 

