Course Description

This graduate level course introduces Monte Carlo methods for optimization, estimation and learning, including: Importance sampling; Sequential importance sampling; Markov chain Monte Carlo (MCMC) sampling techniques including Gibbs samplers, Metropolis/Hastings and various improvements; Simulated annealing; Exact sampling techniques; Convergence analysis; Data augmentation; Cluster sampling, such as Swendsen-Wang and SW-cuts; Equi-energy and multi-domain sampler; and Mapping the energy landscapes.

Prerequisites

- Stat 202B Matrix Algebra and Optimization.
- People who didn't take 202B before may still take this class by asking for a PTE# as long as they have background on matrix algebra, probability theory, and programming skills. So far the course had graduate students from a wide range of departments: Statistics, CS, EE, Mechanical Eng., Civil Eng., Bio-Eng., Economics, Management in business school, Urban planing, Politic Science, Social Science, Geophysics, Physics, Chemestry, ...

Textbooks

The lectures will be based on the following book draft.


Instructors

- Prof. Song-Chun Zhu, sczhu@stat.ucla.edu, 310-206-8693, office BH 9404, Office Hours: Monday 3:30-5:00pm.
- Teaching assistant Mitch Hill, mkhill@ucla.edu, Office hours: Thursday 3:00-5:00pm at MS 8141 TA room.

Grading Plan: 4 units, letter grades

The grade will be based on four parts

1. Homework 20%
2. 2 homework 20%
3. 3 small projects 45%
4. Project 1: Importance sampling for counting the number of SAWs in a lattice (15%) 25%
5. Project 2: Exact sampling of Potts model with Gibbs sampler (15%) 25%
6. Project 3: Cluster sampling for Potts model using Swerndsen-Wang method (15%) 25%
7. Final exam 35%

Tentative List of Topics

Chapter 1, Introduction to Monte Carlo Methods

1. Monte Carlo methods in science and engineering
   - Simulation, estimation, sampling, optimization and learning.
2. Topics and issues in Monte Carlo methods

Chapter 2, Sequential Monte Carlo

1. Importance sampling and weighted samples
2. Advanced importance sampling techniques
3. Framework for sequential Monte Carlo
   - (selection, pruning, resampling, ...)
4. Application on learning log-linear/Gibbs models
5. Application: particle filtering in object tracking

Chapter 3, Backgrounds on Markov Chains

1. The transition matrix
2. Topology of transition matrix: communication and period
3. Positive recurrence and invariant measures
4. Ergodicity theorem

Chapter 4, Metropolis methods and its variants

1. Metropolis algorithm and the Hastings’s generalization
2. Special case: Metropolized independence sampler
3. Reversible jumps and trans-dimensional MCMC

Chapter 5 Gibbs sampler and its variants

1. Gibbs sampler
2. Generalizations:
   - Hit-and-run, Multi-grid, Generalized Gibbs, Metropolized Gibbs
3. Data association and data augmentation
4. Slice sampling

Chapter 6 Clustering sampling

1. Ising/Potts models
2. Swendsen-Wang and clustering sampling
3. Three interpretations of the SW method

Chapter 7 Langevin Dynamics
1. Hamiltonian Monte Carlo
2. Langiven dynamics used in machine learning
   Gibbs Reaction and Diffusion equations, Alternative Back-propagation

Chapter 8 Convergence analysis
1. Monitoring and diagnosing convergence
2*. Contraction coefficient
3. Puskin's order
4*. Eigen-structures of the transition matrix
   (Perron-Frobenius theorem, spectral theorem)
5. Geometric bounds
6*. Exact analysis on independence Metropolised Sampler (IMS)
7*. First hitting time analysis and bounds for IMS (paper)
8. Path coupling techniques.
   - Bounds for Gibbs sampler and Swendson-Wang algorithm (paper).
   * discussed in previous Chapters.

Chapter 9 Exact sampling
1. Coupling from the past CFTP
2. Bounding chains

Chapter 10 Advanced topics
1. Equi-energy and multi-domain sampler
2. Wang-Landau algorithm
3. Stochastic gradient
4. Mapping the energy landscape and case studies
5. Comparing the clustering algorithms
6. Landscapes for curriculum learning