## Ch 4 Classic Parsing Algorithms

1. Pure bottom-up: CYK – chart parsing, 1960s
   (Cocke, Younger, Kasami)

2. Pure top-down: Earley-parser, 1970s
   (Earley, Stockle)

3. Recursive/iterative: Inside-outside algorithm, 1990s
   (Lori, Young)

4. Heuristic: Best-first Chart Parsing, 2000s
   (Chaniak, Johnson, Klein, Manning, et al)

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## Chart Parsing in NLP

- **Motivation:** General search methods are not best for syntactic parsing because the same syntactic constituent may be rederived many times as a part of larger constituents due to the local ambiguities of grammar.
- **Basic idea of chart parsing:** Don’t throw away any information. Keep a record --- a chart --- of all the structures we have found.
- **Two types of chart parsing**
  - Passive chart parsing: it is a bottom-up parsing
  - Active chart parsing, by introducing the agenda, say, agenda-driven chart parsing
    - Bottom-up active chart parsing
    - Top-down active chart parsing
    - The agenda is used to prioritize constituents to be processed, implemented as
      - a stack to simulate depth-first search (DFS)
      - a queue to simulate breadth-first search (BFS)
      - a priority queue to simulate best-first search (FoM)
What is a chart?

- A chart is a form of well-formed substring table [Partial parse graph],
  - It plays the role of the memo-table as in DP,
  - It keeps track of partial derivations so nothing has to be rederived

- Formally, charts are represented by directed graphs $G = \langle V, E \rangle$
  - For an input sentence with $n$ words, $V = \{0, 1, 2, \cdots, n\}$, and the $i$-th word is marked by two nodes in $V$, say, node $i - 1$ and node $i$.
  - Each edge $e \in E$ characterizing a completed or partial constituent spanning a group of words, say, $e = (\text{start}, \text{finish}, \text{label}, \text{found}, \text{tofound}) \in E$
    - label is a nonterminal node in the grammar, say, LHS of a certain rule
    - found is a part of RHS of label which explains words from start to finish
    - tofound is the remainder of beside the found part
  - Active edge: tofound is not empty
  - Inactive edge (passive edge): tofound is empty

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The fundamental rule for combining active and passive edges

- An active edge $e_1 = (i, j, VP, DV, NP, PP)$:
  $$VP \to DV \cdot NPP$$

- An passive edge $e_2 = (j, k, NP, DetN, \emptyset)$:
  $$NP \to Det N.$$

- The fundamental rule:
  $$VP \to DV \cdot DetN \cdot PP$$

- We get an active edge $e_3 = (i, k, VP, DV, DetN, PP)$:
  $$PP \to P \cdot NP.$$

- We get a passive edge (terminal) $e_4 = (i, l, VP, DV, DetN, PP, \emptyset)$:
  $$VP \to DV \cdot DetN \cdot PP$$
What is an agenda?

- An agenda is a data structure that keeps track of the things we still have to do. Simply put, it is a set of edges waiting to be added to the chart.

- The agenda determines in what order edges are added to the chart
  - Stack agenda for depth-first search
  - Queue agenda for breadth-first search
  - Priority queue agenda for best-first search

- The order of elements in agenda is decided by the figures of merit (FOM) of elements, which is one of the keys to design an efficient parsing algorithm.

1, CYK algorithm: an example
Pure bottom-up parser: Cocke–Younger–Kasami (CYK) algorithm

- CYK algorithm is a type of bottom-up passive chart parsing algorithm.

- **The goal:** to determine whether a sentence can be generated by a given context-free grammar (say, recognition) and, if so, how it can be generated (say, parse tree construction). The context-free grammar must be in Chomsky normal form (CNF).

- The worst case running time of CYK is \( O(n^3 |G|) \) where \( n \) is the length of input sentence and \( |G| \) is the size of grammar.
  - The drawback of all known transformations into CNF is that they can lead to an undesirable bloat in grammar size. Let \( g \) be the size of original grammar, the size blow-up in the worst case may range from \( g^2 \) to \( 2^{3g} \), depending on the used transformation algorithm.

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**CYK algorithm**

- **Input:** a sentence \( X = w_1 \cdots w_n \) and the grammar \( G \) with \( S \) being the root.
  - Let \( w_{ij} = w_{i+1} \cdots w_{j} \) be the substring of \( X \) of length \( j \) starting with \( w_i \). Then, we have \( X = w_{1n} \).
- **Output:** verify whether \( S \Rightarrow X \), if yes, construct all possible parse trees.
- **The algorithm:** for every \( w_{ij} \) and every rule \( R \in G \), it determines if \( R \Rightarrow w_{ij} \) and the probability if necessary.
  - Define an auxiliary 4-tuple variable for each rule \( R_b \in G \):
    \[ v_b = (k, \text{probability}, \text{pointerLeft}, \text{pointerRight}) \]
  - CYK table with the entries \( V[i,j] \) storing the auxiliary variables of the rules which can explain substring \( w_{ij} \).
  - Start with substrings of length 1: \( w_{ij} = w_i, 1 \leq i \leq n \) set \( V[1, i] = \{ (k, \text{Prob}(R_b[w_i]), \text{NULL}, \text{NULL}) | R_b \Rightarrow w_i, R_b \in G \} \)
  - Continue with substrings of length \( j = 2, 3, \ldots, n - i + 1 \)
    - For \( w_{ij} \), consider all possible two-part partitions \( w_{ij} = w_{i1}w_{j} \) where \( 1 \leq i \leq j \leq n \).
    - For \( V[i,j] \), consider all possible two-part partitions \( V[i,j] = (k, \text{Prob}(R_b[w_i]), v_{i1}, v_{j}) | R_b \Rightarrow w_{i1}, R_b \in G \)
- **The algorithm has three nested loops each of which has the range at most 1 to \( n \).**
  - With each loop, it check all the rules. So, the worst case of running time is \( O(n^3 |G|) \).
2. Bottom-up passive chart parsing

- Basic algorithm flow: Scan the input sentence left-to-right and make use of CFG rules right-to-left to add more edges into the chart by using the fundamental rule.

**Grammar:**
1. S → NP VP
2. NP → DET ADJ N
3. NP → DET N
4. NP → ADJ N
5. VP → AUX V NP
6. VP → V NP

**Lexicon:**
- the... DET
- large... ADJ
- can... AUX, N, V
- hold... N, V
- water... N, V

Sentence: 3 The 3 large 2 can 3 can 4 hold 3 the 4 water 7
3. Bottom-up active chart parsing

- Algorithm flow:
  - (1) Initialize chart and agenda
    Chart = empty, Agenda = {passive edges for all possible rules for all the words}
  - (2) Repeat until agenda is empty
    • (a) Select an edge from agenda in terms of DFS, BFS or best-first search, etc.
      \[ e = (\text{start}, \text{finish}, \text{label}, \text{found}, \text{to find}) \]
    • (b) Add the selected edge \( e \) to the chart in position (\text{start, finish}) if it is not on the chart
    • (c) Use the fundamental rule to combine the selected edge \( e \) with other edges from the chart, and then add all obtained edges on the agenda
    • (d) If the selected edge \( e \) is a PASSIVE one, say, \( to \text{found} = \emptyset \), then look for grammar rules which have \text{found} as the first symbol on the RHS, say, \( r \to \text{found} V_{\text{remaining}} \). For each \( r \), build active edge \( e' \) and add it on the agenda


An example of bottom-up active chart parsing

Mia danced

Chart

Agenda

4. Top-down active chart parsing (Earley parser)

- **Top-down vs. Bottom-up active chart parsing**
  - Bottom-up chart parsing checks the input sentence and builds each constituent exactly once. Avoid duplication of effort!
  - But bottom-up chart parsing may build constituents that cannot be used legally, as the example shown in previous slide.
  - By working bottom up, the algorithm reads the rules right-to-left, and starts with the information in passive edges.
  - Top-down chart parsing is highly predictive. Only grammar rules that can be legally applied will be put on the chart.
  - By working top-down, the algorithm reads the rules left-to-right and starts with the information in active edges.
Top-down active chart parsing (Earley parser)

- Algorithm flow:
  - (1) Initialize chart and agenda
    Chart={passive edges for all possible rules for all words}, Agenda={root rules}
  - (2) Repeat until agenda is empty
    - (a) Select an edge from agenda in terms of DFS, BFS or best-first search, etc.
      $e = (\text{start}, \text{finish}, \text{label}, \text{found}, \text{tofound})$
    - (b) Add the selected edge $e$ to the chart in position $(\text{start}, \text{finish})$ if it is not on the chart
    - (c) Use the selected rule to combine the selected edge $e$ with other edges from the chart, and then add all obtained edges on the agenda
    - (d) If the selected edge $e$ is a ACTIVE one, say, tofound $\neq \emptyset$, then look for grammar rules which have the forms $r = \text{tofound} \rightarrow V_1 \cdots V_m$ for each $r$; build active edge $e'$ and add it on the agenda
      $e' = (\text{finish}, \text{finish}, r, \emptyset, V_1 \cdots V_m)$
  - (3) succeed if there is a passive edge $e = (0, n, S, \text{found}, \emptyset)$, where $S$ is the root node in the grammar.

Example of Early Parser

<table>
<thead>
<tr>
<th>Production</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-&gt;A B C</td>
<td>a1b1c1</td>
</tr>
<tr>
<td>S-&gt;D E F</td>
<td>a1b1c2</td>
</tr>
<tr>
<td>A-&gt;a1</td>
<td>a1b2c1</td>
</tr>
<tr>
<td>A-&gt;a2</td>
<td>a1b2c1</td>
</tr>
<tr>
<td>B-&gt;b1</td>
<td>a1b2c2</td>
</tr>
<tr>
<td>B-&gt;b2</td>
<td>a1b2c2</td>
</tr>
<tr>
<td>C-&gt;c1</td>
<td>a2b1c1</td>
</tr>
<tr>
<td>C-&gt;c2</td>
<td>a2b1c1</td>
</tr>
<tr>
<td>D-&gt;a1</td>
<td>a2b1c2</td>
</tr>
<tr>
<td>D-&gt;a2</td>
<td>a2b1c2</td>
</tr>
<tr>
<td>E-&gt;b1</td>
<td>a2b2c1</td>
</tr>
<tr>
<td>E-&gt;b2</td>
<td>a2b2c1</td>
</tr>
<tr>
<td>F-&gt;c1</td>
<td>a2b2c2</td>
</tr>
<tr>
<td>F-&gt;c2</td>
<td>a2b2c2</td>
</tr>
</tbody>
</table>
Earley-Stolcke parsing algorithm

<table>
<thead>
<tr>
<th>State set 0</th>
<th>State set 1</th>
<th>State set 2</th>
<th>State set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted</td>
<td>Completed S-&gt;A. B C</td>
<td>Completed S-&gt;A. B C</td>
<td>Completed S-&gt;A. B C</td>
</tr>
<tr>
<td>oS-&gt;A. B C</td>
<td>S-&gt;D. E F</td>
<td>S-&gt;D. E F</td>
<td>S-&gt;D. E F</td>
</tr>
<tr>
<td>oA-&gt;a1</td>
<td>Predicted B-&gt;b1. E-&gt;b1.</td>
<td>Predicted C-&gt;c1. F-&gt;c1.</td>
<td>o-&gt;.S.</td>
</tr>
<tr>
<td>oA-&gt;a2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oD-&gt;a1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oD-&gt;a2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The input is a1 b1 c1

 Modifications of the Earley-Stolcke parsing algorithm

| E1->A B C | E2->D |
| A->a1     | D->d |
| A->a2     |      |
| B->b1     |      |
| B->b2     |      |
| C->c1     |      |
| C->c2     |      |

The input is a1 d b2 c1
## Modifications of the Earley-Stolcke parsing algorithm

<table>
<thead>
<tr>
<th>State set 0</th>
<th>State set 1</th>
<th>State set 2</th>
<th>State set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ $\rightarrow$ E1</td>
<td>Predicted</td>
<td>$\phi$ E1 $\rightarrow$ A B C</td>
<td>$\phi$ A $\rightarrow$ a1</td>
</tr>
<tr>
<td>$\phi$ $\rightarrow$ E2</td>
<td>Predicted</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$ E2 $\rightarrow$ D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
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## Modifications of the Earley-Stolcke parsing algorithm

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<th>State set 0</th>
<th>State set 1</th>
<th>State set 2</th>
<th>State set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>o.E1</td>
<td>Scanned</td>
<td>Shifted</td>
<td></td>
</tr>
<tr>
<td>A-&gt;.a1</td>
<td>A-&gt;a1</td>
<td>B-&gt;.b1</td>
<td></td>
</tr>
<tr>
<td>A-&gt;.a2</td>
<td>A-&gt;a2</td>
<td>B-&gt;.b2</td>
<td></td>
</tr>
<tr>
<td>o.E2</td>
<td>Scanned</td>
<td>Completed</td>
<td></td>
</tr>
<tr>
<td>E2-&gt;.D</td>
<td>D-&gt;.d</td>
<td>D-&gt;.d</td>
<td></td>
</tr>
<tr>
<td>D-&gt;.d</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Stat 232B Stat modeling and inference, S. C. Zhu**
5. Grammars for Events

- UsingWD → ArriveWD UseWD LeaveWD
- ArriveWD → arrivewd
- UseWD → TakeWater
- UseWD → TakeBucket
- TakeWater → benddown1 standup
- TakeBucket → stretchhand drawbackhand
- LeaveWD → leavewd
- BendDown → Pickup
- BendDown → TieString
- Pickup → benddown2 standup
- TieString → benddown2 standup

AOGs representation for Events

AOGs for Events

Event 2:

BendDown

PickUp

TieString

Bend

Stand

Up

Bend

Stand

Down

Up

The parsing process

UsingWD

ArriveWD

TakeWater

Bend

Stand

Down

Up

LeaveWD

TakeBucket

Stretch

Hand

Drawback

Hand

PickUp

TieString

Bend

Stand

Down

Up

The parsing process

The parsing process

UsingWD

ArriveWD

TakeWater

BendDown1

StandUp

StretchHand

DrawbackHand

LeaveWD

UseWD

TakeBucket

BendDown2

StandUp

BendDown

PickUp

TieString

BendDown2

StandUp

BendDown2

StandUp

The parsing process

Stat 232B  Stat modeling and inference,  
S.C. Zhu
The parsing process


The parsing process
UsingWD

ArriveWD

BendDown

TakeWater

TakeBucket

Stand Up

Stretch Hand

Drawback Hand

Bend Down1

Up

Bend Down2

Up

TieString

PickUp

LeaveWD

Stand Up

Bend Down1

Up

Bend Down2

Up

The parsing process

The parsing process

UsingWD

ArriveWD

UseWD

TakeWater

Bend Down1

Stand Up

Bend Down2

Stand Up

Drawback Hand

Drawback Hand

LeaveWD

TakeBucket

Stretch Hand

PickUp

TwistString

aw

bd

su

sh

dh

lw

6. Best-first chart parsing

- The agenda uses a priority queue to keep track of partial derivations.
- The order of constituents in agenda is calculated based on some figures of merit of constituents which measure the likelihood that the constituents will appear in a correct parse, rather than simply popping constituents off of a stack (which simulates DFS) or a queue (which simulates BFS).

Ideally, the objective is to pick the constituent that maximizes the conditional probability: \( p(N_{j,k}^i | t_{0,n}) \).

The key to best-first chart parsing is how to estimate it.

---

**Ideal figures of merit**

\[
p(N_{j,k}^i | t_{0,n}) = \frac{p(N_{j,k}^i, t_{0,n})}{p(t_{0,n})} = \frac{p(N_{j,k}^i, t_{0,j}, t_{j,k}, t_{k,n})}{p(t_{0,n})} \\
= \frac{p(t_{0,j}, N_{j,k}^i, t_{k,n}) p(t_{j,k} | t_{0,j}, N_{j,k}^i, t_{k,n})}{p(t_{0,n})} \\
\approx p(t_{0,j}, N_{j,k}^i, t_{k,n}) p(t_{j,k} | N_{j,k}^i, t_{k,n}) = p^{\text{in}}(N_{j,k}^i) p^{\text{out}}(N_{j,k}^i)
\]

where the inside probability and the outside probability

Inside probability includes only words within the constituent

Outside probability includes the entire context of the constituent
Simple designs for the figures of merit

(1) Straight inside probability: 
\[ p^{in}(N^i_{j,k}) = p(t_{j,k} | N^i) \]
which prefers shorter constituents to longer ones, resulting in a “thrashing” effect due to omit 
\[ p^{out}(N^i_{j,k}) \] and \( p(t_{0,n}) \).

(2) Normalized per-word inside probability:

(3) Trigram estimate:
\[
p(N^i_{j,k} | t_{0,n}) = \frac{p(N^i_{j,k} | t_{0,n})}{p(t_{0,n})} \approx \frac{p(t_{j,k} | t_{0,j}, t_{0,n}) p(t_{0,j} | t_{0,n})}{p(t_{j,k} | t_{0,j}, t_{0,n})} = p(t_{j,k} | t_{0,j}, t_{0,n}) \frac{p(t_{0,n})}{p(t_{j,k} | t_{0,j}, t_{0,n})}
\]
Assume 
\[ p(N^i_{j,k} | t_{0,j}, t_{0,n}) \approx p(N^i_{j,k}) = p(N^i) \]
and use a trigram model
for 
\[
p(t_{j,k} | t_{0,j}, t_{0,n}) \approx p(t_{j,k} | t_{0,j-2}, t_{0,j-1}) = \prod_{k=2}^{n} \frac{p(t_{0,j-k}, t_{0,n})}{p(t_{j,k} | t_{0,j}, t_{0,n})}
\]
we have,
\[
p(N^i_{j,k}) = p(t_{j,k} | t_{0,j-2}, t_{0,j-1}) \]
--- a planar rectangle in 3-space

7, FoM example: scene parsing

--- a planar rectangle in 3-space

Six grammar rules which can be used recursively

\[
\begin{align*}
r_1 & : S ::= S \\
r_2 & : A ::= \text{line} \\
r_3 & : A ::= \text{mesh} \\
r_4 & : A ::= \text{nesting} \\
r_5 & : \text{instance} ::= \text{rectangle} \\
r_6 & : \text{cube} ::= A_1 A_2 A_3
\end{align*}
\]

Two configuration examples
Bottom-up detection (proposal) of rectangles

Each rectangle consists of two pairs of line segments that share a vanish point.

Top-down / bottom-up inference

Red ones are in chart, blue ones are in agenda
Each grammar rule is an assembly line and maintains an Open-list and Closed-list of particles.

A particle is a production rule partially matched, its probability measures an approximated posterior probability ratio.

Open-list or agenda:

\[
\begin{align*}
    \Psi_1 & \rightarrow \cdots \rightarrow \psi_3 \\
    \Psi_4 & \rightarrow \cdots \rightarrow \psi_5 \\
    \Psi_6 & \rightarrow \cdots \rightarrow \psi_7 \\
    \Phi & \rightarrow \cdots \rightarrow \psi_8
\end{align*}
\]

Example of top-down / bottom-up inference:
Results
(Han and Zhu, 05)

Edge map

Rectangles inferred

Likelihood model based on primal sketch

\[ \Lambda = \Lambda_{sk} \cup \Lambda_{nsk}, \]
\[ \Lambda_{sk} = \bigcup_{k=1}^{N} \Lambda_{sk,k}, \]
\[ \Lambda_{nsk} = \bigcup_{m=1}^{M} \Lambda_{nsk,m}, \]
\[ \Lambda_{nsk,m_1} \cap \Lambda_{nsk,m_2} = \emptyset, m_1 \neq m_2. \]

\[ p(I_{sk} \mid C) \propto \exp\left\{ - \sum_{(x,y) \in \Lambda_{sk,k}} \frac{(I(x,y) - B_k(x,y))^2}{2\sigma^2} \right\} \]

\[ p(I(C) \mid G) = \frac{1}{Z} \exp\left\{ - \sum_{k=1}^{N} \sum_{(x,y) \in \Lambda_{sk,k}} \frac{(I(x,y) - B_k(x,y))^2}{2\sigma^2} - \sum_{m=1}^{M} \sum_{i=1}^{n} (\beta_m, h_i(\Lambda_{nsk,m})) \right\} \]
Synthesis based on the parsing model
Synthesis based on the parsing model

Parsing rectangular scenes by grammar
How much does top-down improve bottom-up?

In the rectangle experiments:

![ROC Curve for Rectangle Detection Using Bottom-up only and Bottom-up/Top-down](image)

**Han and Zhu, 2005-07**

8, Chart-agenda parsing with And-Or graphs
The And-Or graph is a recursive structure. So, consider a node A.
1. any node A terminate to leaf nodes at a coarse scale (ground).
2. any node A is connected to the root.

Starting the $\alpha/\beta/\gamma$ channels when they are applicable --- an optimal scheduling problem

An example: human faces are computed in 3 channels
Human faces in real scenarios

$\alpha$-channel

$\beta$-channel

$\gamma$-channel

Hierarchical modeling and $\alpha$, $\beta$ and $\gamma$ computing

1. Each node has its own $\alpha$, $\beta$ and $\gamma$ computing processes.

2. How much does each channel contribute?

$\gamma$: $p(\text{face} \mid \text{parents})$

$\alpha$: $p(\text{face} \mid \text{compact image data})$

$\beta$: $p(\text{face} \mid \text{parts})$
**α processes for the face node**

α-channels: \( p(\text{face} \mid \text{compact image data}) \)

**γ processes for the face node (when it’s α and β is off)**

γ-channels: \( p(\text{face} \mid \text{parents}), \text{predicting} \)  

α-channels of some parents are on
In general: recursive $\alpha$, $\beta$ and $\gamma$ channels

Deriving the log-probabilities

$$p_{\gamma}^* = \arg \max_{\theta \in \Omega} p(p|q)p(I_{A|p|q})$$

$$p(I_{A|p|q}) = q(I_{A|p|q})$$

$$p(I_{A|p|q}) = q(I_{A|p|q})$$

Deriving the log-probabilities

Log probabilities becomes the weights for each node:

\[
\begin{align*}
\alpha_A &= \log \frac{p(I_{A_1} | t_A)}{q(I_{A_1})} \\
\beta_A &= \sum_{i=1}^{2} \log \frac{p(I_{A_i C_i} | t_{C_i})}{q(I_{A_i C_i})} + \log p(X(C_1), X(C_2)) \\
\gamma(A) &= \log \frac{p(I_{A_{1p}} | x_{1p})}{q(I_{A_{1p}})} + \log p(X(A) | X(P))
\end{align*}
\]

\[\alpha_{\text{channel: head-shoulder}}\]
\( \alpha \)-channel: head-shoulder
\( \alpha \)-channel: face

\( \alpha \)-channel: face
\[ \alpha \text{-channel: face} \]

\[ \alpha \text{-channel: eye} \]
α–channel: nose
\( \alpha \)-channel: nose

\( \alpha \)-channel: mouth
$\alpha-$channel: mouth
All $\alpha$ channels

keep things and throw away stuffs by integrating $\alpha$, $\beta$ and $\gamma$ channels

Integrating $\alpha$, $\beta$ and $\gamma$ channels

Threshold $s$
Integrating $\alpha$, $\beta$ and $\gamma$ channels
Integrating $\alpha$, $\beta$ and $\gamma$ channels

Integrating $\alpha$, $\beta$ and $\gamma$ channels
Integrating $\alpha$, $\beta$ and $\gamma$ channels

Integrating $\alpha$, $\beta$ and $\gamma$ channels

Integrating $\alpha$, $\beta$ and $\gamma$ channels


S. C. Zhu
Integrating $\alpha$, $\beta$ and $\gamma$ channels

Integrating $\alpha$, $\beta$ and $\gamma$ channels
Integrating $\alpha$, $\beta$ and $\gamma$ channels
Integrating $\alpha$, $\beta$ and $\gamma$ channels

Performance improvement

red for $\alpha$, blue for $\alpha+\beta$, green for $\alpha+\gamma$, cyan for $\alpha+\beta+\gamma$ channels