Single-View 3D Scene Reconstruction and Parsing by Attribute Grammar

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Abstract—In this paper, we present an attribute grammar for solving two coupled tasks: i) parsing an 2D image into semantic regions; and ii) recovering the 3D scene structures of all regions. The proposed grammar consists of a set of production rules, each describing a kind of spatial relation between planar surfaces in 3D scenes. These relations are directly encoded with a hierarchical parse graph representation where each graph node indicates a planar surface or a composite surface. Different from other stochastic image grammars, the proposed grammar augments each node (or production rule) with a set of attribute variables to depict scene-level global geometry, e.g. camera focal length, or local geometry, e.g. surface normal, contact lines between surfaces. These geometric attributes impose constraints between a node and its off-springs in the parse graph. Under a probabilistic framework, we develop a Markov chain Monte Carlo method to construct a parse graph that optimizes the 2D image recognition the 3D scene reconstruction simultaneously. We evaluated our method on both public benchmarks and newly collected datasets . Experiments demonstrate that the proposed method is capable of achieving state-of-the-art 2D semantic region segmentation and single-view 3D scene reconstruction.

Index Terms—3D Scene Reconstruction, Scene Parsing, Attribute Grammar.

1 INTRODUCTION

The goal of computer vision, as coined by Marr [32], is to compute what and where, which correspond to the tasks of recognition and reconstruction respectively. The former is often posed as parsing an image in a hierarchical representation, e.g., from sketches, semantic regions, objects, to scene categories. The latter recovers 3D scene structures, including camera parameters [55], surface normals and depth [21], and local Manhattan world [6]. While the recognition and reconstruction problems are usually addressed separately or sequentially in the literature, it is mutually beneficial to solve them jointly in a tightly coupled framework for two reasons.

• 2D image parsing is capable of providing semantic contextual knowledge for pruning the uncertainties during 3D modelling. For example, if two neighbor pixels are classified the same label (e.g. building), it is likely that they are projections of the same 3D plane. In addition, semantic region labels, e.g. building or groundplane, often provide strong prior on surface normal.

• 3D reconstruction can provide additional geometric information to boost recognition. In the literature, there have been a number of efforts that utilizes geometry to help region segmentation [31], [16], objection detection [21], visual tracking [39] or event classification [49], etc.

To couple the two tasks, we propose an attribute grammar as a unified representation, which augments levels of geometric attributes (e.g., camera parameters, vanish points, surface normal etc.) to the nodes in the parse graph. Thus the recognition and reconstruction tasks are solved in a joint parsing process simultaneously. Fig. 1 shows a typical parsing result with seven planar surfaces plus a sky region and a high-quality 3D scene model.

Fig. 1. A typical result of our approach. (a) Input image overlaid with detected parallel lines; (b) surface normal map where each color indicates a unique normal orientation; (c) synthesized images from a novel viewpoint; and (d) depth map (darker is closer).

1.1 Overview of our approach

We consider outdoor urban scenes that may contain multiple local Manhattan worlds (LMW) or ‘mixture Manhattan world’ [45], where, for example, buildings are composed of multiple planar surfaces and touch the ground on contact lines. In contrast to the widely used Manhattan world assumption [6], this paper considers a more general scenario that, the adjacent surfaces of a building may not be orthogonal to each other (see the main building in Fig. 1). Curved surfaces are approximated by piecewise linear splines. The surface is further decomposed into super-pixels and edge elements. These representational units can be naturally organized in a hierarchical parse graph with the root node being the scene and terminal nodes being the edges and super-pixels.
Including focal length of camera, and Cartesian Coordination System defined by Manhattan frames.

Fig. 2. Parsing an image using attribute grammar. Left: **global geometric attributes** are associated with the root node (scene) of the parse graph, including focal length of camera, and Cartesian Coordination System defined by Manhattan frames. Right: **local geometric attributes**, such as surface normals and vanishing points (VPs) associated with a surface, or multiple vanishing points for a building. 

\( R_1, \ldots, R_5 \) are the five grammar rules for scene decomposition.

Different from the widely studied appearance attributes of scenes in the vision literature [48] [49] [56], our interest is in the **geometric attributes** for all the nodes in the parse graph. An edge segment has its associated vanishing point, and a super-pixel has a surface normal, a planar facet of a building has two vanishing points, and a building has 3 vanishing points, and finally the whole scene shares a set of camera parameters (focal length etc.). We amount these geometric attributes to the parse graph as is shown in Fig. 2. In this attribute parse graph, attributes of a node can be inherited by its offspring, and thus impose geometric constraints in the hierarchy. These constraints are expressed as additional energy terms in the parsing algorithm so as to maintain consistency in the hierarchy. Consequently, the parsing and reconstruction problems are solved in a tightly coupled manner. This attribute parse graph is different from, and can be integrated with, other scene parsing problems, e.g., fine-grained scene classification [48] that uses appearance attributes “cast sky”, “yellow field” etc.

To construct the attribute parse graph, we define an attribute grammar which is a 5-tuple: \( G = (V_T, V_N, S, \mathbf{R}, \mathbf{P}) \). The set of terminal nodes \( V_T \) include surface fragments or superpixels, the non-terminal nodes \( V_N \) include planar surfaces, composite surfaces, building block and Manhattan world, the root node \( S \) is the scene, and \( \mathbf{R} \) is the set of production rules, and \( \mathbf{P} \) is the probability associated with the rules. Each node \( a \in V_T \) (or \( a \in V_N \)) is associated with set of geometric attributes.

We observe that a few production rules (or compositions) are capable of explaining most of the outdoor urban scenes. We construct 5 production rules which are quite generic for urban scenes. Each rule \( A \rightarrow A_1, \ldots, A_k \) represents a certain spatial arrangement between the children surfaces \( A_1, \ldots, A_k \), and imposes constraints on the attributes of \( X(A) \) and \( X(A_1), \ldots, X(A_k) \).

These composition rules compete with each other to interpret the input image in a recursive way, which results in a parse graph as a valid interpretation of the scene. The parse graph includes both appearance models for 2D segmentation and geometric models for 3D reconstruction.

We formulate the inference of attribute parse graph from a single image in a probabilistic framework. The state space is the set of all possible attribute parse graphs with large structural variations. To efficiently sample this complex state space, we adopt the Data-Driven Markov Chain Monte Carlo paradigm [47]. In particular, our inference method starts with an initial parse graph constructed by a greedy method, and then simulates a Markov Chain in the state space by a set of diffusion-jump dynamics [2]. During the initialization stage, we utilize a heuristic search procedure for camera calibration, and introduce a belief propagation method to obtain region labelling which leads to an initial parse graph. During the following sampling stage, we introduce five dynamics that are paired with each other to exploit the joint solution space periodically, which can guarantee nearly global convergence [47].

A short version of this work appeared in CVPR’2014 [31] and we extend it in both modelling and inference. In **modelling**, [31] uses geometric attributes to impose hard constraints that switch on or off the corresponding probability models, whereas this work uses both semantic and geometric attributes to impose soft constraints to define a set of calibrated energies models, resulting in a more flexible model. In **inference**, this work introduces a stage-wise MCMC sampling method which is more effective than [31] in terms of accuracies and convergences. Moreover, we collect and annotate a new image dataset of 950 images, and evaluate both methods on it. Results show that the newly proposed method achieved much better performance in terms of convergences and reconstruction/labelling accuracies.
1.2 Related Works

Our work is closely related to the following four research streams in computer vision.

Semantic scene labelling has been widely studied to deal with appearance variations, low-resolution and semantic ambiguities. A popular choice is the Conditional Random Fields [27] model that describes qualitative contextual relations between region segments. Such relations are proved to be helpful in the recognition of outdoor objects. Choi et al. [5] further studied a 2D context model to guide detectors and produced a semantically coherent interpretation for the given image. Felzenszwalb and Vekser [10] applied the dynamic programming method for pixel labelling of 2D scenes with simple “tiered” structure. These methods formulate scene labelling as a pixel-wise labelling problem which however ignores the hierarchical and recursive composition relations between regions. In contrast, our method models semantic regions using a hierarchical parse graph which can be used to understand the input image at different levels, from pixels to regions to scene layout.

Single-View 3D modelling has been extensively studied in previous literature. Han and Zhu [16] studied a generative model for reconstructing objects and plants from a single-view. Hoiem et al. [20] explored rich geometric features and context information to recognize normal orientation labels of 2D regions, and Heitz et al. [18] further proposed to recognize geometric surface labels and semantic labels simultaneously in a supervised way. Gupta et al. [13] considered 3D objects as blocks and inferred their 3D properties such as occlusion, exclusion and stability. However, these methods were built on the classification of 2D segmentation, and thus did not directly reconstruct 3D or infer depth values. Mobahi et al. [33] reconstructed a single view by extracting low rank textures on building façade. Saxena et al. [42] and Haene et al. [15] ever studied a fully supervised model to build mappings between informative features and depth values. Schwing et al. [44] presented an exact inference method (i.e. branch-and-bound) for single-view indoor scene reconstruction. Pero et al. formulated the 3D reconstruction of room in a Bayesian framework and proposed a sampling method for inference [36], [37], [38]. Ladicky et al. [26] proposed a discriminatively trained boosting method for estimating surface normal.

The above mentioned methods tried to recover global 3D scene without an explicit representation of camera model and 3D geometric structures. In contrast, our method jointly formulates 2D region labelling problem and 3D reconstruction problem with an attribute grammar model and explores the joint solution by constructing an optimal hierarchical parse graph representation. The obtained graph not only directly encodes high-quality 3D scene model but also provides interpretable decompositions of the input image in both 2D and 3D that are helpful to solving higher level perception problems, e.g. object activity recognition.

Joint Recognition and Reconstruction has been investigated for a number of computer vision tasks. Haene et al. [15] presented a continuous-discrete formulation for jointly solving scene reconstruction and labelling of images of multiple views. Ladicky et al. [25] proposed to train a depth-wise classifier for each class, used to predict semantic classes and depth maps for a single image. Their method requires groundtruth depth maps for training. Carbral et al. [3] tried to recover planar-wise 3D scene model from panorama images of office areas, which extended the previous works by Xiao et al. [50].

The other studies include jointly solving object recognition and object modelling. Haene et al. [14] proposed to learn 3D shape priors from surface normals which has been proved to be very successful. Hejrati et al. [19] proposed to synthesize 2D object instances from 3D models and used the instances to help solve object recognition task. Schwing et al. [43] introduced a method for recovering 3D room layout and objects simultaneously. Xiao et al. presented a supervised method for localizing 3D cuboids in 2D images [52]. They also introduced a benchmark [51] for joint Structure-from-Motion and Object Labelling. Payet and Todorovic [34] proposed a joint model to recognize objects and estimate scene shape. Zhang et al. [54] proposed to reconstruct a room using Panoramic images by exploiting both object parsing (e.g. table detection) and scene geometry (e.g. vanishing points).

Moreover, joint formulation has also been applied for simultaneous tracking and reconstruction [24] [53], joint object recognition and reconstruction [1] [29], floor-plan layout estimation [30] and video reconstruction [24]. Our work follows the same methodology and contributes an attribute grammar for joint image labelling and scene reconstruction. The developed techniques can be applied to the above mentioned joint tasks as well.

Scene grammar. Koutsourakis et al. [23] proposed a shape grammar to reconstruct building faades. The proposed model focused on rectifying faade images but not recovering 3D geometry. Han and Zhu [17], Zhao and Zhu [56] and Pero et al. [35] built generative scene grammar models to model the compositionality of Manhattan structures in the indoor scenes. Furukawa et al. [11] studied the reconstruction of Manhattan scenes from stereo inputs. In contrast, we relax the Manhattan assumption and generalize the scene grammar model to handle more complex and cluttered outdoor environment. We contribute a hierarchical representation for urban scene modelling and augment it with both semantic and geometric attributes.

In comparison with the literature, the paper makes the following contributions:

1) We present a grammatical model with geometric attributes that tightly couples the image parsing and 3D scene reconstruction tasks.
2) We develop a stage-wise sampling inference method that is capable of exploiting the constrained space efficiently.
3) In experiments on both public datasets and our self-collected datasets, our method achieves considerably better performances than the existing methods in terms of both 2D parsing and 3D reconstruction.

1.3 Paper Organization

The rest of this paper is organized as follows. We will introduce a hierarchical scene representation in Section 2, present a probabilistic scene grammar model in Section 3, and discuss the inference algorithm in Section 4. We report the experiment results in Section 5, and conclude this paper with a discussion in Section 6.

2 Representation: Attribute Hierarchy

Given an input image, our goals include: i) recovering the scene geometry structure, ii) partitioning the scene into planar surfaces and iii) reconstructing the planar-wise 3D scene model. These goals can be unified as solving the optimal parse graph with
geometric and semantic attributes. In this section, we overview the hierarchical entities of outdoor scene and their attributes.

Camera Parameter We assume that there is no distortion, no skew, and that the principle point coincides with the image center. Thus we need to estimate camera focal length \( f \) and camera viewing directions. The viewing directions can be described by Manhattan frames since we consider Manhattan type urban scenes. We subtract principle point from the coordinate of each pixel to facilitate representation.

2.1 Geometry Attributes from Edge Statistics

In man-made scenes, texture gradients and edges are not arbitrarily oriented, but reflect camera orientations with respect to the scene and surface layout in 3D space. Hence, we can extract the geometric attributes from edge statistics.

2.1.1 Attributes of Edges and Parallel Lines

In the pinhole camera model, a family of parallel lines, i.e. sharing the same 3D direction, in the 3D space project to straight edges that all point to the same point on the image plane, i.e. the vanishing point. Thus each line segment in the image has two geometric attributes:

- A vanishing point \( (x_i, y_i) \) in the image plane to which an edge points to. This can be directly obtained by clustering oriented edges based on their directions in 2D image plane.
- A 3D direction \( \theta = (x_i, y_i, f) \) of edges or parallel lines in the 3D scene space where \( f \) is the camera focal length.

As Fig. 3 illustrates, it follows from perspective geometry the ray from the camera position \( O \) to \( (x_i, y_i) \) is parallel to the families of parallel lines as well. Therefore, its direction is the unit vector by normalizing the triple vector \( (x_i, y_i, f) \).

2.1.2 Attributes of Local Manhattan World

Outdoor urban scene often contains a mixture of local Manhattan worlds [6]. Each local Manhattan world is a block of well aligned buildings with three sets of orthogonal parallel lines. Each set of parallel lines has a vanishing point \( (x_i, y_i) \) and 3D direction \( \theta = (x_i, y_i, f) \). We refer to the rays from camera origin \( O \) to the vanishing points as the Manhattan axes. Thus each local Manhattan world has the following geometric attributes:

- A Manhattan frame with three orthogonal Manhattan axes \( \{ (x_1, y_1, f), (x_2, y_2, f), (x_3, y_3, f) \} \).
- An estimated focal length \( f \) following [4]

\[
  f^2 = -(x_i, y_i) \cdot (x_j, y_j), \quad i \neq j \in \{1, 2, 3\}. \tag{1}
\]

This follows the orthogonal condition that

\[
  (x_i, y_i, f) \cdot (x_j, y_j, f) = 0. \tag{2}
\]

It is worth noting that this estimated focal length will be propagated to the scene node in the attribute parse graph. The equation \( (x_i, y_i, f) \cdot (x_j, y_j, f) = 0 \) poses consistency conditions among the attributes of Manhattan axes.

2.2 Attribute Planar Representation

In parallel to the edges, lines/parallel lines, and Manhattan structures, the region-based hierarchy comprises of three representations: surface fragments, planar surfaces, and composite surfaces. Fig. 4 summarizes the attribute planar representation.

We augment every hierarchical entity with both semantic attributes and geometric attributes. The semantic attribute of an entity, e.g. planar surface, is simply its semantic category. In this work, we consider a few semantic categories for outdoor scenes, including “building”, “tree”, “ground”, “sky” and “other”. A composite surface might include two or more than two categories. Geometric attributes are used to describe the spatial properties of the hierarchy, which will be introduced in the rest of this section.

2.2.1 Geometric Attributes of Surface Fragment

We assume that each super-pixel in images is the projection of a surface fragment in space. A super-pixel is a small region of pixels that are connected and share similar appearance features, and often

<table>
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<tr>
<th>Representation</th>
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<td>Surface Fragment</td>
<td><img src="image1" alt="Surface Fragment Example" /></td>
<td>Surface Normal; VPs;</td>
</tr>
<tr>
<td>Planar Surface</td>
<td><img src="image2" alt="Planar Surface Example" /></td>
<td>Surface Normal; Contact Line; Manhattan Axes; 2 VPs;</td>
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<tr>
<td>Composite Surface</td>
<td><img src="image3" alt="Composite Surface Example" /></td>
<td>Surface Normal; Contact Spline; Manhattan Axes; K VPs;</td>
</tr>
<tr>
<td>Scene</td>
<td><img src="image4" alt="Scene Example" /></td>
<td>Focal Length; Vanishing Points; Manhattan Frames;</td>
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have the same semantic label. Since these super-pixels often correspond to regions in buildings or marked road/highways/ground, which have edges or texture gradients, from which we can extract short edges and estimate which vanishing points they belong to.

As Fig. 3 illustrates, each super-pixel has two geometric attributes:

- Two vanishing points: \( \{(x_1, y_1), (x_2, y_2)\} \) and thus two Manhattan axes \( \{v_1 = (x_1, y_1, f), v_2 = (x_2, y_2, f)\} \).
- A surface normal direction which is the cross-product of the two Manhattan axes \( n = (x_1, y_1, f) \times (x_2, y_2, f) \).

For each superpixel, we extract its vanishing points and surface normal from local edge statistics, which might not be necessarily accurate. To improve robustness against noises, these statistics will be pooled together in bottom-up process and propagated to other nodes in the attribute parse graph. For a super-pixel that does not contain sufficient number of edges, its surface normal will be inferred from surrounding scene context, i.e. top-down process in the parse graph.

### 2.2.2 Geometric Attributes of Planar Surface

We group spatially connected super-pixels into planar surfaces based on two types of features. i) Appearance features. We extract color and texture features to train a supervised classifier and assign a region to a few categories, e.g. ‘building’, ‘tree’, etc. ii) Geometry features. Superpixels in the same planar region should share the same surface normal. Both features are used in the iterative parsing and process to form planar surfaces.

Each planar surface has three geometric attributes:

- Two vanishing points: \( \{(x_1, y_1), (x_2, y_2)\} \) and thus two Manhattan axes \( \{v_1 = (x_1, y_1, f), v_2 = (x_2, y_2, f)\} \).
- Normal direction. As aforementioned, surface normal is simply the cross-product of the two Manhattan axes.
- A contact line and thus its 3D relative depth. The surface plane will intersect with other planes and form the contact lines. For example, Fig. 4 shows three planar surfaces of the building and their ground contact boundaries which can be approximated by straight lines respectively.

The contact lines may be occluded (e.g. between a building facade and the ground) or blurred (line between two surfaces of the building). Fortunately this can be solved by calculating the intersection line between adjacent surface planes, which usually points to one of the Manhattan axes associated with the surface planes. These geometric attributes are sufficient to reconstruct a planar-wise 3D scene model [21].

### 2.2.3 Geometric Attributes of Composite Surface

A composite surface consists of several planar surfaces that are physically connected. These surfaces might not belong to the same Manhattan frame. A composite surface has set of geometric attributes that pose consistency constraints between its children nodes in the parse graph. Its geometric attributes include:

- All vanishing points and surface normal of its planar surfaces.
- Contact lines between adjacent surfaces.
- A linear spline fit of the contact lines with the ground.

As planar surfaces, e.g. building facade, are usually occluded by foreground objects, e.g. vehicles and trees, and their boundaries to the ground plane are often partially visible. In Section 4 we shall introduce a robust method for estimating contact splines under these severe occlusions.

### 2.3 Geometric Attributes of Scene

The whole scene will pool over the geometric attributes from its components. As it is shown in Fig. 2, the root node \( S \) has the following geometric attributes.

- Camera parameters are shared by all nodes in the parse graph. Note that our model can be extended to reason other camera parameters, including skew, and optical center etc. 
- \( m \) Manhattan frames \( \{(x_{ij}, y_{ij}, f), i = 1, 2, ..., m, j = 1, 2, 3\} \) for each local Manhattan world.

These global geometric attributes are used to constrain the geometric attributes of the entities in the parse graph. For example, the number of possible normal directions for planar surfaces are determined by the number of Manhattan axes detected for the global scene. In contrast, the past methods [22] [21] usually fix the number of surface normal orientations during inference.

### 3 Probabilistic Scene Grammar

In this section, we introduce a probabilistic treatment of the proposed attribute scene grammar.

#### 3.1 Attribute Scene Grammar

Attribute grammar was firstly proposed by Han et al. in [17]. We extend it to model hierarchical scene representations in both 2D images and 3D scene space.

An attribute grammar is specified by a 5-tuple: \( G = (V_N, V_T, S, R, P) \), where \( V_N \) is a set of non-terminal nodes, \( V_T \) is a set of terminal nodes, \( S \) is the root node for the whole scene, \( R \) is a set of production rules for spatial relationships, and \( P \) is a probability for the grammar.

These production rules can be recursively applied to generate a hierarchical representation of the input scene, namely Parse Graph. A parse graph is a valid interpretation of the input 2D image and the desired 3D scene. A grammar generates a large set of valid parse graphs for one given image of the scene.

**Terminal Nodes** We partition the input image into a set of superpixels and use them as terminal nodes. Each superpixel is the projection of a surface fragment in space. We denote all terminal nodes as \( V_T = \{a, X(a)\} \), where \( X(a) \) denotes a set of attribute variables.

**Non-Terminal Nodes** are sequentially produced by merging terminal nodes or other non-terminals with grammar rules. Each node represents a planar surface or composite surface in space. There is one root node for the whole scene, i.e. \( S \), and five production rules. Every non-terminal node in parse graph can be decomposed into children nodes or grouped with other nodes to form parent nodes by applying the above grammar rules.

We denote all non-terminal nodes as \( V_N = \{(S, X(S)), (A, X(A))\} \) where \( S \) denotes the root node for the whole scene, \( A \) non-terminal node and \( X(A) \) the attributes of \( A \). Fig. 5 illustrates these five rules and Fig. 2 shows one parse graph that is capable of generating the input image.

**Global and Local Attributes** Each node is associated with a number of attributes, which are either globally or locally defined.

Global attributes are defined for the root node \( S \) and inherited by all graph nodes. \( X(S) \) includes i) a list of possible categories (e.g., ‘building’) that appear in the input image, denoted as \( C \); ii) geometric attributes, including the camera focal length \( f \) and Manhattan frames detected in the input image. Formally, we have
Local attributes are defined over properties of intermediate nodes, e.g., surface normal. These attributes are usually inherited from the global attributes and thus should be consistently assigned. Fig. 2 illustrates global geometric attributes in the left panel and local geometric attributes in the right panel. Semantic attributes are not included in the figure. Both global or local attributes are used to impose constraints to obtain valid parse graphs.

### 3.2 Probabilistic Formulation for 3D Scene Parsing

We utilize a hierarchical parse graph to explicitly encode the attribute hierarchy (introduced in Section 2) for joint recognition and reconstruction purposes. In particular, terminal nodes are able to form planar configuration in imaging plane or surface normal map of the input image; the parse graph with geometric attributes can be used to derive a full 3D scene model for reconstruction purpose.

Formally, let $G$ denote the parse graph to solve, $A$ all attributes in $G$. Given an input image $I$, we compute a world scene interpretation $W$ in a joint solution space

$$ W = (A, G) $$

The optimal solution $W^*$ can be obtained by maximizing a posterior probability (MAP):

$$ P(W | I) \propto \exp \{ -|V_N| - \lambda^{pA} E(I, G, A) \} $$

where $|V_N|$ indicates the number of non-terminal nodes. We use the first item to encourage compact parse graphs. $\lambda^{pA}$ is a weight constant.

The energy $E(I, G, A)$ is defined over the hierarchy of $G$, indicating how well $G$ can generate the input image $I$. Let $r(A)$ indicates the grammar rule used at $A$. We have,

$$ E(I, G, A) = \sum_{A \in V_N} \beta_{r(A)} E^i(I, X(A)|r(A)) $$

where $r(A) \in [1..5]$ indicates the grammar rule associated with $A$, $\beta_{r(A)}$ is a weight constant that is dependent on $r(A)$. The energy term $E^i(I, X(A)|r(A))$ is associated with the nonterminal node $A$ and conditioned on the corresponding grammar rule $r(A)$.

### TABLE 1
Definition of Grammar rules and their geometric attributes.

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<th>Rules</th>
<th>Notations</th>
<th>Geometric Attributes</th>
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<td>$R_1$: Layout</td>
<td>$A \rightarrow (A_0, A_1, ..., A_m)$</td>
<td>$X(A) = (f, m, \theta_i, \theta_j, l_k, C)$</td>
</tr>
<tr>
<td>$R_2$: Siding</td>
<td>$A \rightarrow (A_1, A_2)$</td>
<td>$X(A) = (\theta, M, l_k, c)$</td>
</tr>
<tr>
<td>$R_3$: Affinity</td>
<td>$A \rightarrow (A_1, A_2)$</td>
<td>$X(A) = (\theta, M, c)$</td>
</tr>
<tr>
<td>$R_4$: Mesh</td>
<td>$A \rightarrow (A_1, A_2, ..., A_n)$</td>
<td>$X(A) = (\theta, M, v_1, v_2, c)$</td>
</tr>
<tr>
<td>$R_5$: Instance</td>
<td>$A \rightarrow a$</td>
<td>$X(A) = (\theta, M)$</td>
</tr>
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</table>

Table 1 summarizes the definitions, e.g., geometric attributes, of all grammar rules. In the rest of this subsection, we introduce the definitions of five grammar rules.

#### 3.2.1 Grammar Rule $R_1$: Layout

The **Layout** rule $R_1 : A \rightarrow (A_0, A_1, ..., A_m)$ states that a planar surface $A_0$ is supporting $n$ entities. In this work we assume that all stuffs (objects, building, etc) in the scene are standing on ground. $A_0$ indicates the ground region in images (e.g. grass, road, side walk etc.), and $A_1, ..., A_n$ indicates the $n$ children surfaces or composite surfaces produced by other grammar rules. Fig. 5 illustrates the use of $R_1$, which merges two building blocks/surfaces and the ground. The rule $R_1$ is used to generate the root node $S$.

The geometric attributes of $S$ include both global attributes and local attributes defined over its children nodes. The former includes, a list of possible categories, camera focal length $f$ and $m$ Manhattan frames. Each Manhattan frame includes three axes in space that are orthogonal to each other. The later includes the normal directions of children surfaces, e.g., $\theta_i$ for $A_0$, and the contact lines between $A_0$ and each of the $m$ entities, denoted as $l_k$. Formally, we have $X(S) = (f, m, M_i, \theta_{ij}, l_k, C), i, k = 1, ..., n$, where $\theta_{ij}$ represents one of the normal orientations in the $i^{th}$ child node, $C$ a list of category labels.

We use continuous splines $l_k$ to represent contact boundaries between $A_0$ and $\{A_k\}$, which are assumed to be piece-wise linear. Fig. 6 illustrates four typical scenes where contact splines are highlighted in red. A piece-wise linear spline consists of several control points and straight lines between them. Each straight line corresponds to the contact boundary of a planar region. In urban images, a contact line is usually parallel to one of the parallels falling in the support region. This gives rise to a useful observation: if we can detect local edges in the given planar region and cluster these edges to parallel families, the direction of a contact line can be simply determined. With this observation, we will develop an effective search algorithm for discovering contact splines in Section 4.

We define the energy function for $R_1$ from two aspects. Firstly, the normal direction of the surface $A_0$ and other children surfaces should be as distinct as possible. This is different from the previous works [13] [22] which assume orthogonality between connected surfaces. Secondly, contact lines are likely to go through VPs that...
have edges falling in \(A_k\). Thus, we have,
\[
E^\epsilon(\mathbf{I}, X(A)|R_1) = \sum_{i,j} D^\cos(\theta_i, \theta_j) + \lambda_{by} \sum_{l \in I_k} \min \{D^\cos(l, v)\}
\]
\[\forall v \in M, M \in X(A_k)\]
where \(l\) indexes the line segment in the spline \(\overrightarrow{t_k}\), \(M\) the Manhattan world in \(X(A_k)\), \(D^\cos\) the cosine distance between two directions or two straight lines in 3D space. Note that \(v\) indicates one of the Manhattan axes in the Manhattan world associated with \(A_k\).

### 3.2.2 Grammar Rule \(R_2\): Siding

The **siding rule** \(R_2 : A \rightarrow (A_1, A_2)\) states that two planar surfaces or composite surfaces of the same label are spatially connected in the scene. The parent node \(A\) is a composite surface and the child nodes \(A_1, A_2\) could be planar surfaces of composite surfaces. It requires that children surfaces share the same semantic label (e.g. building) but have different normal orientations. These surfaces are usually, but not necessarily, orthogonal with each other.

The attributes of \(R_2\) include \(X(A) = \{(\theta_1, M_1), \overrightarrow{t_k}, c\}\), where \(\theta_i\) is normal direction of the children surface \(A_i\), \(i = 1, 2\), \(M_i\) the Manhattan frame associated with \(A_i\), \(\overrightarrow{t_k}\) the contact line between children surfaces, and \(c\) the semantic label.

The energy function for \(R_2\) is derived from two aspects. Firstly, two siding surfaces should have as distinct normal as possible, which is the case in most of the urban images. Secondly, the contact line of \(A\) is likely to point to the vertical VP, denoted as \(v_0\), as illustrated in Fig. 5. Formally, we have,
\[
E^\epsilon(\mathbf{I}, X(A)|R_2) = \sum_{i \neq j} D^\cos(\theta_i, \theta_j) + \lambda_{ad} \sum_k D^\cos(\overrightarrow{t_k}, v_0)
\]
where \(\lambda_{ad}\) is a weight constant. Taking the production rule \(R_2 : A \rightarrow (A_1, A_2)\) as example, the energy in Eq. (7) shall be minimized if \(A_1\) and \(A_2\) are orthogonal and they are split by a ray in image starting from the vertical VP.

Note that the semantic attributes \(c\) are used as hard-constraints: a graph node of \(R_2\) is only valid when the two children surfaces \(A_1\) and \(A_2\) share the same label.

![Fig. 6. Illustration of piece-wise linear spline model for the contact boundaries of composite surfaces that comprise of groundplane and buildings. Each spline consists of several control points and the straight lines between these points. Note that each straight line correlates with one planar region in the composite surface.](image)

### 3.2.3 Grammar Rule \(R_3\): Affinity

The **affinity rule** \(R_3 : A \rightarrow (A_1, A_2)\) states that two planar surfaces have similar appearance and thus should belong to the same planar surface. The children surfaces \(A_1\) and \(A_2\) should be spatially connected in 3D scene. In practice, since they could be disjoint in image due to occlusions, we allow the grouping of disjoint regions by this rule if they have high affinity in appearance. The attributes of \(A\) are defined as \(X(A) = (\theta, M, c)\) where \(\theta\) is the normal direction, \(M\) the related Manhattan frame, and \(c\) the semantic label, which are shared by the two children surfaces.

The grammar rule \(R_3\) requires that the children surfaces \(A_1\) and \(A_2\) should have the same surface normal. Thus, the geometric attributes serve as hard constraints and we only utilize the appearance information to derive the energy function \(E^\epsilon(\mathbf{I}, X(A)|R_3)\).

The energy function for \(R_3\) include both unary terms and pairwise terms, all of which are defined over superpixel partition of the parent surface \(A\). Let \(s\) and \(l\) index two neighbor superpixels, \(c_s\) the semantic label of superpixel \(s\). We have,
\[
E^\epsilon(\mathbf{I}, X(A)|R_3) = \sum_s \phi_s(c_s) + \lambda_{aff} \sum_{s,t} I(c_s = c_t)
\]
where \(\phi_s(c_s)\) returns the negative class likelihood ratio, and \(I()\) is an indicator function. Like [46], we estimate \(\phi_s(c_s)\) by applying a non-parametric nearest neighbor estimator over training data.

The second term is defined as a Potts/Ising model to encourage homogeneity of labelling.

We estimate surface normal based on edge statistics, as introduced in Section 2. However, if an image region does not contain any local edges, there is no cue to tell its normal direction directly, and we need to infer its normal from the scene context. In Section 4, we shall introduce a robust inference method to deal with these uncertainties.

### 3.2.4 Grammar Rule \(R_4\): Mesh

The **mesh rule** \(R_4 : A \rightarrow (A_1, A_2, A_3, \ldots)\) states that multiple surfaces are arranged in a mesh structure. Children surfaces should be spatially connected to each other and share the same normal direction. In perspective geometry, a mesh structure in image plane can be described by two orthogonal VPs. Formally, the attributes of \(A\) include: \(X(A) = (\theta, M, v_1, v_2, c)\), where \(v_1 = (x_1, y_1, f), v_2 = (x_2, y_2, f)\) are the coordinates of two VPs, \(\theta = v_1 \times v_2\) is the normal direction of \(A\), \(c\) the semantic label. The children surfaces share the same normal direction \(\theta\) with \(A\).

The energy function for \(R_4\) is defined over edge statistics. As Fig. 6 illustrates, straight edges in a mesh region usually merge at two VPs. Let \(\mathcal{E}(A)\) denote the set of local edges in \(A\), and \(\mathbf{l}_j = (x_j, y_j, d_j) \in \mathcal{E}\) an edge at the position \((x_j, y_j)\) with the orientation \(d_j\). Let \(v_i\) denote the image coordinate of the VP \(v_i\). If an edge \(\mathbf{l}_j\) points to \(v_i\), we have \((x_j, y_j) + \lambda_{mes} d_j = v_i\). Thus, we define \(E^\epsilon(\mathbf{I}, X(A)|R_4)\) as:
\[
E^\epsilon(\mathbf{I}, X(A)|R_4) = \sum_{j \in \mathcal{E}(A)} \min_{1,2} \|v_i - (x_j, y_j) - \lambda_{mes} d_j\|^2
\]
where \(i = 1, 2\). This least square energy term is minimized while all edges in the mesh region exactly point to one of the two VPs, i.e. \(v_1\) or \(v_2\).

### 3.2.5 Grammar Rule \(R_5\): Instance

An instance rule \(R_5 : A \rightarrow a\) instantiates a terminal node, i.e. a superpixel or a surface fragment, to image representations, including both texture appearances and edge segments. Fig. 5 illustrates how the grammar rule \(R_5\) links a non-terminal node to two image representations: histogram of oriented gradient (HoG) and straight edge map.
The potential $E'(I, X(A)|R_S)$ is defined over two aspects: i) the appearance of individual pixels in the region of $A$ should be homogeneous; ii) the directions of local straight edges should be consistent with the Manhattan frame assigned or inherited from the parent nodes of $A$. Let $I_i$ and $I_k$ denote two neighbor pixels in region $A$, we have:

$$E'(I, X(A)|R_S) = \sum_{i,k} g(I_i, I_k) + \lambda_{ins} \sum_{l \in E(A)} \min \{D^\cos(l, v) \}
$$

where $g(I_i, I_k)$ returns the negative confidences of two pixels being homogeneous, $\lambda_{ins}$ is a weight constant. The model $g(I_i, I_k)$ is directly estimated by the superpixel partition method [40] with both HoG features and edge features. The second term is used to encourage that all edges in $A$ should be parallel to one of the Manhattan axes in $X(A)$.

Fig. 2 shows an exemplar parse graph generated by the proposed grammar. Each grammar rule describes a kind of spatial relationship, e.g., $R_1$ for supporting, $R_2$ for being co-block, $R_3$ and $R_4$ for being co-planar. These simple rules are capable of producing a large number of tree-structure representations whereas only a portion of them are valid. It is worth noting that the tree structures are augmented to be parse graphs by linking nodes in the same layer that are spatially connected. This graph representation encodes both 2D appearance and 3D geometric properties of the hierarchical scene entities (as introduced in Section 2).

4 Inference

Our inference algorithm aims to construct an optimal parse graph by sequentially applying the grammar rules to maximize a posterior $P(W|I)$. This task is challenging because: a) the optimal parse graph does not have a pre-defined structure; b) the attribute constraints over attribute hierarchy are of high-order.

We develop a stage-wise method to solve the optimal parse graph, which includes three major stages. Firstly, we introduce an efficient algorithm to calculate camera parameters, i.e., the geometric attributes of the root node $S$ and the parameters $A$ throughout inference. Note that the semantic attributes of $S$ (i.e., category labels) are manually set. Secondly, we solve the region labelling to optimization by minimizing the energy function of Eq. (8), w.r.t superpixel labels $c_i$. Eq. (8) is a typical MRF type energy function that consists of a unary term and a regularization term of Potts/Ising prior. It can be efficiently solved by the loopy belief propagation (LBP) method [9]. We use the results of region labelling to initialize the desired parse graph. Finally, we introduce a data-driven Monte Carlo Markov Chain (DDMCMC) method to sample the posterior probability $P(W|I)$.

Algorithm 1 summarizes the proposed inference algorithm. It includes two bottom-up computation steps and an iterative sampling step that simulates the Markov Chain with a set of dynamics. The first two steps are used to narrow the search space and thus speed up the sampling procedure. We introduce these steps in the rest of this section.

4.1 Bottom-up Computation: Calibration by Heuristic Search

We develop a stochastic heuristic search procedure to solve the optimal camera focal length and Manhattan frames. We first utilize the hough transform based voting method by Li et al. [28] to detect families of parallel lines and their associated vanishing points (VPs). Next, we apply Eq. (1) over every pair of parallel families to estimate the camera focal length, by assuming they are orthogonal to each other. Let $S$ denote the number of pairs of parallel families. We associate a binary variable to every pair, denoted as $d_i \in [0, 1]$. $d_i = 1$ if the $i^{th}$ pair of families is orthogonal otherwise $d_i = 0$. Thus, we can solve camera focal length by minimizing the following objective:

$$\min_{f_i(d_i)} \frac{1}{S} \sum_{i=1}^{S} ||d_i f_i - \hat{f}||$$

where $f_i$ is the estimation of the camera focal length from the $i^{th}$ pair of parallel families (by assuming they are orthogonal and applying Eq. (1)), $\hat{f}$ denotes the estimation of the camera foal length.

To optimize Eq. (11), we introduce a heuristic search procedure. It starts with initializing at random $\{d_i\}$ followed by two iterative steps. Step 1: estimate focal length $f_i$ from the $i^{th}$ pair if $d_i = 1$ and average over all estimations to get $\hat{f}$; Step 2: assign $d_i$ to be 1 with the probability of $1/(1 + \exp([-f_i - \hat{f}]))$. We iterate these two steps until convergence.

4.2 Bottom-up Computation: Belief Propagation for Region Labelling

The goal of this step is to assign every superpixel of the input image to one of the five semantic labels, including ‘sky’, ‘building’, ‘ground’, ‘trees’ and ‘other’. This is equal to estimate the optimal superpixel label assignment so as to minimize the energy function of Eq. (8) w.r.t. the superpixel labels $c_i$.

We estimate the unary term in Eq. (8) as follows. Each superpixel is described using 20 different features, including shape, location, texture, color and appearance [46]. We first extract these features for training images and store with their class labels. Next, we associate a semantic label with a training superpixel if more of the superpixel overlaps with the ground truth segment mask of that label. In the following, we compute class likelihood ratio for each superpixel in the testing image, using the nearest neighbor estimator [46]. Last, the labelling of a testing image is obtained by simply assigning each superpixel to the class that maximizes the likelihood.

We use the efficient loopy belief propagation algorithm by Felzenszwalb et al. [9] to finalize the labelling. We consider the min-sum algorithm that works by passing messages around the graph defined by the connected grid of superpixels. Each message is a vector of dimension given by the number of possible labels, 5 in this work. Since the smoothing term $\varphi_{<c,t,>} is semi-metric, the propagation algorithm can converge in $O(|C|NT)$ time where $|C|$ is the number of labels, $N$ is the number of superpixels, and $T$ is the number of iterations. Each iteration of the message updates is very fast since we only have $|C| = 5$ candidate labels. We fix the maximal iteration number to be 10.

4.3 Iterative MCMC sampling

Following the computations of camera calibration and region labelling, we design a data-driven Markov Chain Monte Carlo sampling algorithm (DDMCMC) [47] to search for the optimal parse graph. It starts with an initial parse graph that includes one root node and a set of terminal nodes, as illustrated in Fig. 7 (a). In the following, we further merge neighbor terminal nodes
Dynamics 1-2: birth/death of nonterminal nodes are used to create or delete a nonterminal node and thus transition the current parse graph \( G \) into a new graph \( G' \) as illustrated in Fig. 7.

The proposals for creating a nonterminal node was made by first selecting at random one of the four grammars, \( R_1, \ldots, R_4 \). Next, for the selected grammar rule, we obtain a list of candidates that are plausible according to the predefined constraints. Taking \( R_2 \) as example, two children nodes should i) have different normals; ii) be spatially connected and iii) be assigned to the same semantic label. Each candidate in this list is represented by its energy. Let \( B^k_i \) denote the \( i^{th} \) candidate for the grammar rule \( R_k \), its energy is \( E^i(I, X(B^k_i)|R_k) \). The list is as follows,

\[
L^b = \{ B^k_1, E^i(I, X(B^k_1)|R_k), i = 1, 2, \ldots \}
\]  

The proposal probability for selecting \( B^k_i \) is calculated from the weighted list,

\[
Q(W \rightarrow W') = 1 - \frac{E^i(I, X(B^k_1)|R_k)}{\sum_j E^i(I, X(B^k_j)|R_k)}
\]

Similarly, we obtain another set of candidate nodes to delete based on their energies,

\[
L^d = \{ D^k_i, E^i(I, X(D^k_i)|R_k), i = 1, 2, \ldots \}
\]

The proposal probabilities for deleting the node \( D^k_i \) is calculated as follows:

\[
Q(W \rightarrow W') = \frac{E^i(I, X(D^k_1)|R_k)}{\sum_j E^i(I, X(D^k_j)|R_k)}
\]

Dynamics 3-4: Merge/split regions are used to re-label the superpixels around the boundaries between different semantic regions (e.g. "sky" and "building"). These jumps are used together to polish the image labelling by the bottom-up computation in subsection 4.2. Fig. 7 (c) illustrates one typical example.

We obtain the list of candidate proposals for the merge/split dynamics as follows. Firstly, we take the superpixels on the boundaries of two neighbor regions as graph nodes. These superpixels are usually with big ambiguities and the discriminative methods [9] do not necessarily work well. Secondly, we link all perpixels are usually with big ambiguities and the discriminative methods [9] do not necessarily work well. Secondly, we link all

\[
L^m = \{ \text{CCP}_i, h(\text{CCP}_i|W), h(\text{CCP}_i|W'), i = 1, 2, \ldots \}
\]

The proposal probability for selecting the \( i^{th} \) candidate is defined as follows:

\[
Q(W \rightarrow W') = \frac{h(\text{CCP}_i|W')/h(\text{CCP}_j|W')}{\sum_j h(\text{CCP}_j|W')/h(\text{CCP}_j|W)}
\]

**Dynamic 5: Switching Geometric Attributes** We design two diffusion dynamics to change the geometric attributes of graph nodes. As aforementioned, the geometric attributes of the root node, including camera focal length and Manhattan frames, are calculated and fixed throughout the inference. The geometric attributes of nonterminal nodes mainly include their respect normals and contact splines.
Swiching Normal $\theta$. In local Manhattan world, every normal direction corresponds to a Manhattan axe or a family of parallel lines. To determine the normal of a surface region, we simply determine for every edge in this region its vanishing point of parallel family [28] and accumulate all assignments to find two mostly used orthogonal VPs. These two VPs can be used to determine surface normal. Fig. 3 illustrates the geometric relations between surface normal and local edge statistic. Since edge directions include many noises, we use the estimated surface normal to initialize the geometric attributes of graph nodes at the beginning and refine it in the probabilistic framework. In particular, during inference, we select at random one of the planar surfaces and change its normal randomly. We set the proposal probability to be constant so the acceptance probability is simply based on the posterior probability ratio.

Estimating Contact Spline $I$. This dynamic is used to greedily estimate the contact spline for each composite surface. We take the grammar rule $R_1$: $A \rightarrow (A_0, A_1, A_2, \ldots, A_n)$ for instance to introduce our method for estimating contact spline. As aforementioned, a contact spline consists of control points and straight lines between them, representing the boundary between the children surface $A_i$ and the supporting surface $A_0$. Our method is based on the following observation: a contact line of $A$ is likely to go through one of the VPs associated with $A$.

Let $\mathbf{V}$ denote the set of vanishing points (VPs), $E$ the set of edges with two end points: $<\mathbf{v}_i^1, \mathbf{v}_i^2> \in E$ in the children surface $A_i$. Let $B$ denote the set of boundary points and $b^{ij} \in B$ the point coordinate. Let $\mathbf{v}^i$ denote the VP that the contact line $<\mathbf{c}^{i-1}, \mathbf{c}^i>$ points to. Our goal is to infer $n + 1$ control points $\{\mathbf{c}^i\}$, and search for the associated VP for each of the $n$ contact lines, denoted as $\mathbf{v}^i$. Such two goals can be achieved by minimizing the following function:

$$
\min_{\{\mathbf{c}^i, \mathbf{v}^i\}} \sum_{i,j,k} \text{Dist}(\mathbf{c}^{i-1}, \mathbf{c}^i, \mathbf{v}^j) + \lambda^{bd} \text{Dist}(\mathbf{c}^{i-1}, \mathbf{c}^i, b^{ij})
$$

$$
s.t. \quad \mathbf{v}^i \in \mathbf{V}, b^{ij} \in B, <\mathbf{v}_i^1, \mathbf{v}_i^2> \in E
$$

where the function $\text{Dist}(\mathbf{c}^{i-1}, \mathbf{c}^i, \mathbf{v}^j)$ returns the minimal distance between the point $\mathbf{v}^i$ and the line $<\mathbf{c}^{i-1}, \mathbf{c}^i>$. $\lambda^{bd}$ and $\lambda^{ed}$ are two constants. Eq. (19) minimizes the following three types of distances.

1) $\text{Dist}(\mathbf{c}^{i-1}, \mathbf{c}^i, \mathbf{v}^i)$, the distance between the desired contact line and its associated VP;
2) $\text{Dist}(\mathbf{c}^{i-1}, \mathbf{c}^i, b^{ij})$, the distance between the desired contact line and each of the boundary points, used to minimize the errors of fitting the boundary pixels with the solved spline;
3) $\text{Dist}(\mathbf{v}_i^1, \mathbf{v}_i^2, \mathbf{v}^i)$, the distance between an edge segment in $A_i$ and the desired VP $\mathbf{v}^i$.

In general, Eq. (19) is a NP-hard optimization problem. Fortunately, the feasible space is not huge and thus even an exhaustive search method is computationally acceptable. In order to deal with outliers and noises, we use the RANSAC technique to search for the approximate solution. We always greedily solve the optimal contact spline, in order to reduce the computational complexity of our inference. Fig. 6 shows four exemplar results of our approach. It is worthy noting the ground boundaries could be partially occluded or even fully occluded by objects (e.g. vehicles) or stuffs (tree). The proposed method can predict the correct contact lines because edge statistics from surfaces are used for reasoning.

5 Experiments

In this section, we apply the proposed algorithm to recover 3D model from single-view, and evaluate it in both qualitative and quantitative ways.

5.1 Evaluation Protocols

Datasets. We use four datasets for evaluations. The first one is the CMU dataset collected by Hoiem et al. [22] and we use a subset of 100 images provided by Gupta et al. [13]. Annotations of occlusion boundaries and surface normals are provided. The surfaces are labelled with three main classes: ‘ground’, ‘sky’ and ‘vertical’, and the ‘vertical’ class is further divided into five subclasses: ‘left’, ‘center’, ‘right’, ‘porous’, and ‘solid’. There are only three possible orientations for vertical surfaces. Note that our method associates normal orientations with Manhattan frames and a scene of local Manhattan world might have more than three frames. To utilize these datasets, we arrange the discovered surface normals from left-hand to centroid to right-hand and link them the labels of ‘left’, ‘center’ and ‘right’. We used the first 50 images for training and the rest for testing as [13].

We further collect three datasets from different sources and manually annotates VPs, region labels and surface normal orientations. The first dataset LWM-A consists of 50 images from the collections in [22], and there are 4.6 VPs per image on average. The second dataset LWM-B consists of 50 images from the dataset of EurasianCities in [7] with 4.2 VPs per image on average. The third one LWM-C consists of 950 images selected from the PASCAL VOC [8] and Labelme projects [41]. There are 3.5 VPs per image on average. These three datasets are used for testing only and our model is trained on the CMU dataset.

Model Training. We utilize an empirical study of log-likelihood over training samples to estimate the optimal parameters in the model $p(W|I)$, including the $\lambda$s, $\beta$s and the kernel widths used for the exponential functions. For each of these parameters we empirically quantify its possible values, e.g. 0.1, 0.3, ..., 1 for $\beta$. Our goal is to select the optimal value for each parameter, i.e. the optimal parameter configuration. For a training image, with every possible parameter configuration, we need to simulate a parse graph that is unknown from the provided surface normal map. To do so, in Algorithm 1, we did not call the step 4 for region labelling, and only use the dynamics 1-2 (birth/death of non-terminal nodes) and dynamic 5 (switching geometric attributes). This revised Algorithm 1 usually converges within a hundred of iterations (with dozens of graph nodes). We calculate the log likelihood $\log p(W|I)$ after convergence. Thus, we select the parameter configuration that achieves the maximum log-likelihood. Similar simulation based maximum likelihood estimation (MLE) method has been used in previous works [47] [56].

Since the parse graph is unknown, for each possible parameter configuration we apply the inference method with in Section 4 to simulate a parse graph from the groundtruth surface normal map.

Implementation of Algorithm 1 We resize images so the maximal dimension is 500 pixels and use the method by Ren et al. [40] to partition each image into 200-300 superpixels. We set the maximal iteration numbers to be 2000. It costs 5-6 minutes for Algorithm 1 to converge on a Dell Workstation (i7-4770 CPU@3.4GHZ with 16GB memory).

Baselines. We compare our method to two previous methods: i) the geometric parsing method by Hoiem et al. [22], ii) the method by Gupta et al. in [13]. Both methods can recover the
stage-3: iteration 300 Stage-3: iteration 500 Stage-3: iteration 1000

Fig. 8. Focal length estimation. (a) Input image overlaid with parallel families of edges (colored); (b) focal length estimated by orthogonal pairs of VPs; (c) focal length estimated by non-orthogonal pairs of VPs. The true focal length is 500 (red dotted lines).

The true focal length is 500.

three main geometric classes and the five vertical subclasses. We use the default parameters in their source codes.

We further implement three variants of the proposed method in order to evaluate the effects of individual grammar rules. i) Ours-I, that uses grammar rules $R_1$ (layout), $R_2$(siding), $R_4$(mesh), and $R_5$ (instance) to explore geometric relationships between lines/edges, e.g. orthogonality or co-linear. ii) Ours-II, that uses grammar rules $R_3$ (affinity) and $R_5$ (instance) to explore appearance affinity between regions/superpixels. iii) Ours-III, that uses all grammar rules. All these implementations have to include $R_5$ to get likelihood. In addition, we include the region labelling results of the Belief Propagation algorithm for comparisons, denoted as $BP$.

5.2 Results

Camera Calibration We first demonstrate how the orthogonality conditions of parallel families can be used to estimate camera focal length, as introduced in Section 4.1. We use the image shown in Fig. 8(a), where one vertical VP and four horizontal VPs are detected. Fig. 8(b) plots the estimated focal length (vertical direction) by solving Eq. (1) on four orthogonal pairs of VPs, i.e. the vertical VP and each of the four horizontal VPs. Fig. 8(c) plots the focal length estimated from non-orthogonal pairs of horizontal VPs by solving Eq. (1). The true focal length is 500 for this image, plotted as red dotted lines in both figures.

We can observe that i) in Fig. 8(b), the estimated focal lengths are roughly same (low variance) and the average focal length 510 is quite close to the true value (high accuracy); ii) in Fig. 8(c), in contrast, the estimations are with large variance, and most of them are not close to the true value. Therefore, we can jointly estimate focal length and orthogonality conditions between parallel families. To do so, we use the heuristic search method (see Section 4.1) to minimize Eq. (11).

Qualitative Evaluations Fig. 9 visualizes how Algorithm 1 converges over iterations. There are three main stages, stage-1: camera calibration, stage-2: region labelling and stage-3: iterative MCMC sampling. In the first row of Fig. 9, we plot the input image, and the surface normal maps obtained by the stage-2 and stage-3 (after 100 iterations) of Algorithm 1. In the second row we plot three surface normal maps after 300, 500 and 1000 iterations. The figures are overlaid with contact splines when applicable. We can observe that surface normal maps are continuously refined by the iterative MCMC sampling algorithm. In the third row of Fig. 9 we plot the convergence curve of Algorithm 1, i.e., energy $E(I,G,A)$ w.r.t iterations. Note that we only plot the energy in the Stage-3. We also plot the convergence curve of the previous method [31]. In order to make side-by-side comparisons, we scale the two curves so that they start from the same energy. We can observe that Algorithm 1 converges after 1000 iterations which is a lot faster than [31]. The reasons are two folds: i) the bottom-up computation step for region labelling in Algorithm 1 provides a good initialization to the MCMC sampling process; and ii) the newly introduced five dynamics are more effective than the dynamics used in [31].

Fig. 10 shows some exemplar results of Ours-III on the CMU dataset [22]. In each cell, we plot (a) the input image overlaid with families of parallel lines, where each color indicates one family; (b) the layout partition where each color indicates one planar surface with unique normal; (c) the estimated depth map where darker pixels indicate being closer to the camera and vice versa; (d-e) the synthesized images from novel viewpoints; (f) the depth map estimated by [22]; and (g) the parse graphs created during inference. In Fig. 10 (g) we only show the top levels of the parse graph where each colored rectangle corresponds to one planar surface in subfigure (b) with the same color. Our results are promising considering that only a single viewpoint of the scene is available. Taking the first example for instance, since the far-right building region in purple is occluded by vehicles and trees, none of the previous methods can tell where is the contact line between this facade and the ground. Our approach, however, is able to infer the contact line from the edge statistics extracted from this region. In particular, parallel lines in this region suggest the contact line is likely to go through the VP in green. The estimated contact line in (c) is very accurate.

In addition, one can observe that the image in the second row of Fig. 10 follows the typical Manhattan World assumption, while other images only follow the Mixture Manhattan World assumption as they contain more than 2 horizontal VPs or the horizontal VPs are not orthogonal with each other. For the second
image, both [22] and our method can produce reasonable depth maps. For the other images, however, [22] tends to assign the same depth to the surfaces of 'vertical', whereas our method can still produce high-quality depth maps. These exemplar results well demonstrate how geometric attributes propagate through the hierarchical parse graph to help create accurate 3D models.

Fig. 11 and Fig. 12 show results of our method on the datasets LMW-A and LMW-B, respectively, and compare to the method by Hoiem et al. [22]. In each cell, we show (a) input image overlaid with parallel families; (b) superpixel partition overlaid with VPs; (c) surface normal map by our method; (d) depth map by our method; (e-g) three novel viewpoint synthesized; and (h) depthmap by [22]. All these images do not satisfy the Manhattan assumption. From the comparisons between (d) and (h), we can observe that our method is capable of creating better 3D models.

Fig. 13 show exemplar results of our method on the dataset LMW-C. While the recovered 3D scene models are considerably accurate, these results demonstrate a few drawbacks of the proposed method. Firstly, our current model does not deal with foreground objects, e.g. vehicle in the first image, pedestrian in the second image; Secondly, our method cannot work well for structure-less regions, e.g. tree or plants in the third and the fourth images, that do not include rich geometric regularizations. Thirdly, our method assumes surface regions to be planar-wise which might not be true, e.g. the image in the second row of Fig.13.

We further apply the proposed method Ours-III over a few indoor images and show a few exemplar results in Fig. 14. We plot the input images, layout segmentation and newly synthesized views in columns from left-hand to right-hand. For these scenes, we consider three categories: floor, ceiling, and wall. Other implementation details remain the same as Ours-III. From these results, we can observe that the recovered layout segmentations are very accurate even when there are clutters in front of the scene entities, e.g. walls are occluded by sofas (first image) or tables (third image). The obtained 3D models, however, can be further improved by reconstructing foreground objects, e.g. persons, tables, pillars etc.

Quantitative Results We report the numerical comparisons of the various methods in term of normal orientation estimation and region labelling. For normal orientation estimation, we use the metric of accuracy, i.e., percentage of pixels that have the correct normal orientation label, and average accuracies over
Fig. 11. Exemplar results on LMW-A dataset. For each cell, we show (a) input image overlaid with families of parallel lines; (b) superpixel partition overlaid with vanishing points; (c) obtained surface normal map; (d) estimated depth map; (e-g) newly synthesized views; (h) depth map by Hoiem et al. [22].

Fig. 12. Results on the LMW-B dataset. For each cell, we show (a) input image overlaid with families of parallel lines; (b) superpixel partition overlaid with vanishing points; (c) obtained surface normal map; (d) estimated depth map; (e-g) newly synthesized views; (h) depth map by Hoiem et al. [22].

test images. On the estimation of main geometric classes, i.e., 'ground', 'vertical', and 'sky', both our method and the baselines can achieve 0.98 accuracy or more in terms of normal orientation. Therefore, we focus on the vertical subclasses, like [13]. We discard the superpixels belonging to ground and sky and evaluate the performance of all methods.

Table 2 reports the numerical comparisons on four datasets. From the results, we can observe the following. Firstly, the proposed Ours-III clearly outperforms other baseline methods on all the four datasets. Taking the CMU dataset for instance, the method by Gupta et al. [13] has an average performance of 73.72%, whereas ours performs at 79.53%. On the other three datasets that have accurate normal orientation annotations, the improvements by our method are even more. As stated by Gupta et al. [13], it is hard to improve vertical subclass performance. Our method, however, can improve these two baselines with large margins. Secondly, Ours-III clearly outperforms other two variants, i.e., Ours-I and Ours-II that use less types of grammar rules. These comparisons justify the effectiveness of the proposed joint inference framework. Thirdly, Ours-III has good margins over our previous method [31]. Although [31] follows the same methodology, this work polishes the modelling and inference (see Section 1.2) that improve the final results further.

Table 3 reports the region labelling performance on the four datasets. We use the best spatial support metric as [13], which first estimates the best overlap score of each ground truth labelling and then averages it over all ground-truth labelling. Our method improves the method [13] with the margins of 9.47, 12.02, 9.96, 8.60 percentages on the four datasets, respectively. It is worthy noting that all the three variants of our methods outperform the baseline BP that provides initializations of region labelling. These comparisons show that jointly solving recognition and reconstruction can bring considerable improvements over recognition.
6 CONCLUSIONS

This paper presents an attribute grammar for 3D scene reconstruction from a single view. We introduce five grammar rules to generate a hierarchical image representation for both 2D parsing and 3D reconstruction purposes. The developed inference method can fully exploit the constrained space efficiently by optimizing both the 2D surface layout and the geometric attributes required for creating full 3D scene model. Extensive evaluations on public benchmarks show that our method outperforms other popular methods by achieving state-of-the-art in single-view 3D scene reconstruction.

Our method is currently limited to the reconstruction of background structures, e.g., building, ground, and tree etc. The developed representation and formulations, however, can be extended to parse foreground objects as well, e.g., car, human etc. This is actually equal to jointly solving object detection, scene parsing and 3D scene reconstruction together. Similarly, the 3D position or pose of an object shall be regularized by the global geometric attributes, e.g., camera focal length, camera viewpoint.

This work contributes a generic framework for jointly solving 2D recognition problems, e.g., classification, detection, recognition, tracking, etc., and 3D reconstruction problems, e.g., camera calibration, depth estimation, geo-localization, etc. There are two particular directions to exploit in the future: i) developing new solution for existing joint tasks, e.g., calibration from tracking [12]; ii) motivating novel vision tasks, e.g., jointly solving tracking and geo-localization. We shall push these two directions in the future.

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