Probability Rules

There are 3 rules that allow you calculate more complex probabilities.

RULE 1: Adding Probabilities
For mutually exclusive outcomes (not occuring at the same time), find the probability that outcome A OR outcome B OR outcome C … will occur by adding the individual probabilities.

Example: As a transit planner, you are evaluating the time allocation for a bus schedule during rush hour. You observe that the bus is early 20% of the time and late 35% of the time. What is the probability that the bus is early or late?

.20 + .35 = .55 or 55%

RULE 2: Complementary Probabilities
Because the outcome probabilities in one event will sum to 1, you can calculate the probability of an outcome NOT occurring. What is the probability that the bus will arrive on time?

1 - (probability of being early + probability of being late) = 1 - .55= .45 or 45%

RULE 3: Multiplying Probabilities
For outcomes within a single event, to calculate the probability of outcome A AND outcome B AND outcome C… will occur, you multiply the individual probabilities.

Example: What is the probability that the bus will be late two days in a row? If they are independent it is:

.35 * .35 = .12

Example: The bus service under study takes passengers to the ferry. The ferry departs late 40% of the time regardless of the bus arrival. What is the probability that the bus is late AND the ferry is late?

Event A: bus is late = 0.35
Event B: ferry is late = 0.40
Bus AND Ferry late) = 0.35 * 0.40 = 0.14
There is a 14% probability that both the bus and ferry will be late.

Working with Probabilities

USUALLY if you want the probability of two or more things happening, you multiply their probabilities together. That is, you want Event 1 to happen AND you want Event 2 to happen.

An important consideration for multiple events is whether they are independent or dependent. Two events are independent if the probability of one occurring does not affect the probability of the other. With dependent events, the probability of one event affects the chances of the other occurring.

Suppose I have two boxes. In box 1 I have 10 tickets with the numbers:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

In box 2 I have 26 tickets with the letters:

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z

Suppose you want to draw a "1" (event 1) AND you want to draw an "V" (event 2). Find the probability of each and multiply them together to get the probability of drawing a 1 AND drawing a V.

Since there are 10 slips of paper in the first hat, the probability of drawing a "1" is 1/10. There are 26 slips of paper in the second hat, so the probability of drawing an "V" is 1/26. Thus, the probability of drawing a 1 AND drawing an V is (1/10) * (1/26), which is about 0.0038.

Note that these events are INDEPENDENT. The probability of drawing a "1" did not affect the probability of drawing a "V"

If I put the tickets back in the boxes and then asked "What if you want the probability of drawing a "B" and a "2"?" Would this problem be any different? NO. But what if I left the tickets out of the boxes? YES because their probabilities are changed.

Notice in we want two things to happen and the word "AND" was used. That's usually a hint that you want to multiply the probabilities together. If you had needed only one OR the other to happen -- that is, you needed only one of the two events (notice we used the word "OR" here) -- then you would add the probabilities. For example, suppose you wanted to draw a "2" OR a "3." Then the probability would be the sum of the two probabilities, that is (1/10) + (1/10), which is 0.2. If you wanted to draw an "A" OR a "B," the probability would be (1/26) + (1/26), which is about 0.0769. Note that these combinations are MUTUALLY EXCLUSIVE

If you wanted to draw a "2" OR an "M," you have to be careful. These events are not mutually exclusive -- that is, we could draw both. When we add the probabilities of drawing a "2" (1/10) with the probability of drawing an "M" (1/26), we've counted the probability of drawing a "2" AND an "M" twice, once in the (1/10) and again in the (1/26).

The solution, just so you know, but is NOT part of this course is that you have to subtract it out once. Thus the probability would be the sum of the two probabilities MINUS the product of the probabilities because they are NOT MUTUALLY EXCLUSIVE, that is (1/10) + (1/26) - (1/10)*(1/26), which is about 0.1347. Notice that it is quite a bit more likely that you would draw a "2" OR an "M" than it is that you would draw a "2" AND an M because you can have many possible combinations.
In probability, the words "AND" and "OR" are special, and they usually mean multiply the probabilities (for AND) and add the probabilities (for OR), respectively.