1. Introduction

Definition. PROBABILITY is the study of CHANCE or the likelihood of "something happening. That is, a certain random process is given (such as rolling a die or spinning a roulette wheel), and we want to know the chance of various outcomes. For instance, when you roll a pair of dice, you might ask how likely you are to roll a seven. In statistics, we call the "something happening" an "event."

2. Basic Rules

A. The CHANCE of a particular event is the percentage of time that event is expected to occur if the same random process is repeated over and over under the same circumstances. For example, the chance that "a fair die comes up showing '1' " is 1/6. (Note this definition implies the basic process if replicable; if the process cannot be repeated, using the word "chance" is not correct.)

B. Chances can take values from 0% to 100%. Events that are unlikely will have a probability near 0%, and events that are likely to happen have probabilities near 100%.

C. The chance of a particular event (say rolling a "1" using a fair die) is equal to 100% minus the chance of the event not happening (i.e. 100% minus not rolling a 1). This is a "complement". Where have you seen 100% before? (think Chapter 5)

3. Calculating Basic Probabilities:

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the number of outcomes of interest
---------------------------------- = probability of an event
total possible number of outcomes
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Using this formula requires that you know how total possible outcomes exist. In any probability problem, it is very important to identify all the different outcomes that could occur. For instance, in the introduction about the dice, you must figure out all the different ways the pair of dice could land, and all the different ways you could roll a seven.

A simple example to start: Suppose we have a box with 4 yellow marbles and 6 blue marbles, and we want to find the probability of drawing a yellow marble at random. In this case we know that all outcomes are equally likely: any individual marble has the same chance of being drawn (i.e. no biases here)

To find a basic probability with all outcomes equally likely, we use the fraction shown above.

The outcome of interest? In our example, where we want to find the probability of drawing a yellow marble at random, our outcome of interest is drawing a yellow marble.

What's the total number of possible outcomes? The total number of possible outcomes consists of all ten marbles in the box, because we are equally likely to draw any one of them.

Using our basic probability fraction, we see that the probability of drawing a yellow marble at random is:4/10 Since 4/10 reduces to 2/5, the probability of drawing a yellow marble where all outcomes are equally likely is 2/5. Expressed as a decimal, 4/10 = .4; as a percent, 4/10 = 40/100 = 40%.

Suppose we number the marbles 1 to 10. What is the probability of picking out number 5?

Well, there is only one number 5 marble, and there are still 10 marbles in the box, so the answer is 1 marble (favorable outcome) divided by 10 marbles (all possible outcomes) = 1/10 or 10 percent.

4. Sampling (p. 225)

Here, Freedman introduces you to the concept of sampling or "drawing" at random from a "box". This image will be important through the remainder of the text.

SAMPLING WITH REPLACEMENT -- a situation where the total possible number of outcomes remains the same with every random draw.

SAMPLING WITHOUT REPLACEMENT -- a situation where the total possible number of outcomes can change with every random draw.

Now let's suppose we have two events: first you draw 1 marble from the 10, and then I draw the next marble from the nine that remain. What is the probability that you will draw a blue one first? What is the probability that I will draw a yellow one second?

When you draw the first marble, there are 10 marbles in the box of which 6 are blue, so your probability of drawing a blue one is 6/10 (60 percent) or 3/5. After you draw it's my turn, but now the size of our sample space has changed because there are only 9 marbles left; THIS IS SAMPLING WITHOUT REPLACEMENT, 4 of them are yellow, so the probability that I'll draw a yellow marble is 4/9.

5. Multiplication, Conditional Probabilities and Independence

The multiplication rule: the chance that two (or more) things happen equals the chance that the first happens multiplied by the conditional chance that the second happens given that the first has happened. And you could extend this to the third, the fourth, etc. Here, we're interested in two or more events occurring and we want to know the total probability.

Our previous example involved, Conditional Probabilities and Sampling without Replacement (13.2). Note that the chance of my choosing a yellow marble changed after you picked the blue one. The probability of my selecting a yellow marble was dependent on you selecting the blue marble. You changed my chances.

If we want to know the total probability of your drawing a blue marble and then my drawing a yellow one taken together, we need to use multiplication and multiply the individual probabilities together. So the Probability that you draw a blue marble and then I draw a yellow marble is

3/5 x 4/9 = 12/45 or, reduced, 4/15 or about 26.67%.

Independence is possible when Sampling with Replacement (13.4) and basically when 2 or more events occur, one has no effect on the others occurring and vice versa. In other words, (from page 230) if A happens, it does not change the chance of B happening, etc. To find out the chance that the two or more things happen, you can multiply their unconditional probabilities.

*If you were to draw a marble and then put it back in the box and then I draw a marble, the probabilities would not change for me. That is, what is your chance of drawing a blue marble first? 3/5. Now you put it back in the box, what is my chance of drawing a yellow marble now? 4/10 (or 2/5). If we want to know the total probability of your drawing a blue marble and then my drawing a yellow one taken together-- but with replacement, we still need to use multiplication and multiply the individual probabilities together. So the answer here is 3/5*2/5 = 6/25 or 24%.*

PRINCIPLE: In the situation where outcomes are independent, you multiply unconditional probabilities. If the outcomes are dependent (i.e. situations like sampling without replacement) then multiply conditional probabilities.

PRINCIPLE:

$$\frac{\text{the number of outcomes of interest}}{\text{total possible number of outcomes}} = \text{probability of an event}$$

This is also called the classical probability definition. Another way to interpret probability is as the long-run relative frequency (long-run fraction) of the event.