A. Overview

Given some data, the regression method summarizes the relationship between two variables X and Y by choosing a line that fits the points as closely as possible. Here, there is a strong sense that one of the variables (Y) **depends** on the other (X), contrast this with Ch. 8 & 9 where either could be X or Y.

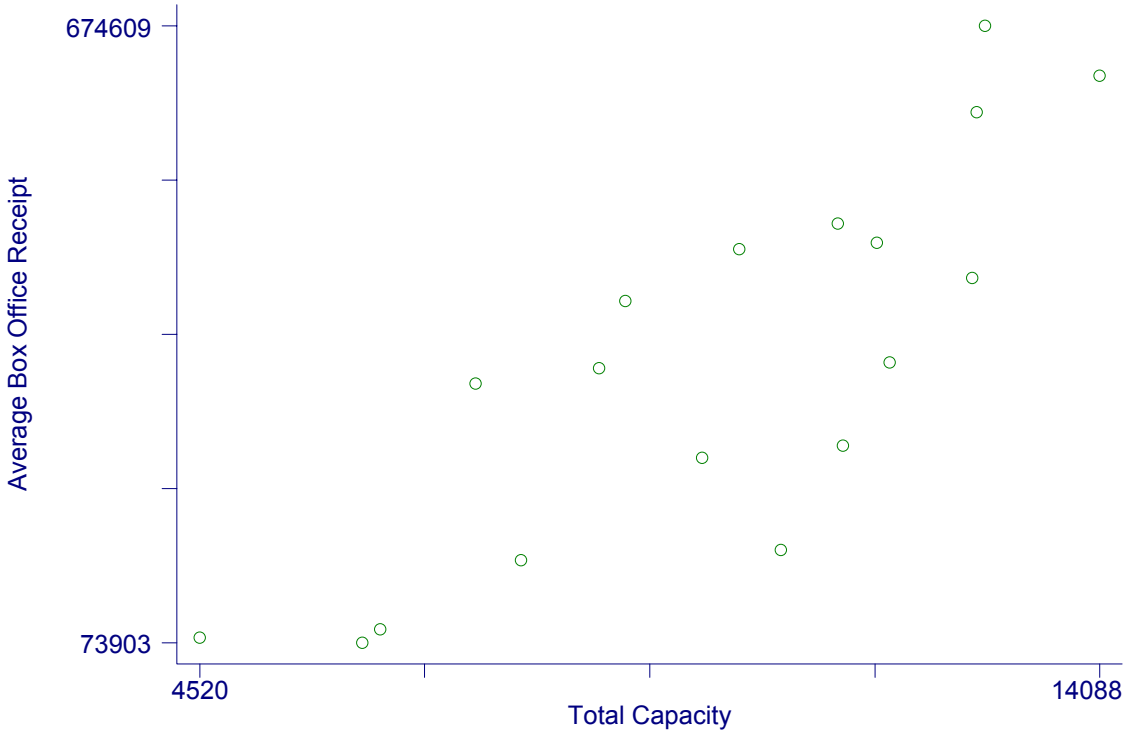
B. INTRODUCTION TO REGRESSION (Chapter 10.1-10.3)

Here we have some data (information) on Broadway shows, their average receipts per show and the capacity of the theater. A question we might be interested in is what is the relationship between capacity and receipts? Do bigger theaters always generate more receipts?

	show	receipts	capacity
1.	Angels in America	326121	7456
2.	Blood Brothers	154064	7936
3.	Cats	346723	11856
4.	Crazy for You	463377	11720
5.	Falsettos	86864	6440
6.	Fool Moon	163802	10696
7.	The Goodbye Girl	429158	12736
8.	Guys and Dolls	457087	10256
9.	Jelly's Last Jam	253951	9864
10.	Kiss of the Spider Woman	406498	9048
11.	Les Miserables	481973	11304
12.	Miss Saigon	625804	14088
13.	Phantom of the Opera	674609	12872
14.	Shakespeare for my Father	78898	4520
15.	The Sisters Rosensweig	340862	8768
16.	Someone Who'll Watch over Me	73903	6248
17.	Tommy	590334	12784
18.	The Will Rogers Follies	265561	11360

The correlation r of theater capacity and receipts is .82. The means and SDs are below:

Variable	Obs	Mean	Std. Dev.	Min	Max
receipts	18	345532.7	187723.8	73903	674609
capacity	18	9997.333	2664.093	4520	14088



From the previous lecture: one of the properties of the correlation coefficient (r) is that it can be used to give us a rough idea of the relationship between two variables.

Values of r near +1 and -1 mean that the two variables are very closely associated. Values of r near 0 suggest that there is almost no relationship between the two. If you look the graph again, r is =.82 which is positive relationship between the variables and it's reasonably strong.

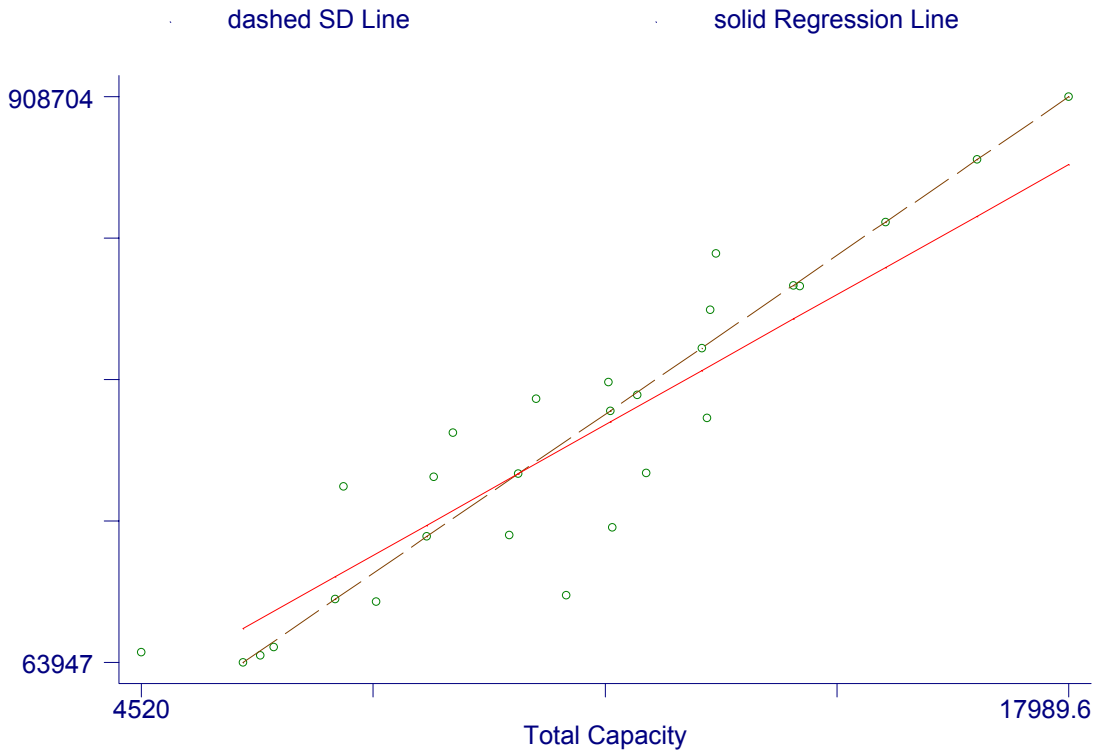
One method of using all of the information is to predict what the receipts will be for a given Capacity level. This method is called the REGRESSION METHOD. You can predict values of your y variable by knowing the averages and the standard deviations and the correlation of both X and Y:

(Average theater capacity) + (one SD of theater capacity) = 9997.333+ 2664.093= 12661.426

So a play that is one SD above the average theater capacity has a theater capacity of about 12,661. Since these two variables (capacity and receipts) appear to be related (a correlation of .82 suggests some relationship) we can ask -- what are the receipts for a play that has above average (one standard deviation above average) theater capacity?

We can quickly answer this by taking the receipts SD of 187723.8, multiply by .82 to get 153933.52 and add this result back to the average receipt of 345532.7 to get 345532.7 + 153933.52 = 499466.22 or near 500,000 dollars.

If you did this method for all possible theater capacity SDs (one, two, three and the numbers in between, you'd get a solid line that looks like:



and you could compare it to an SD line (which is the equivalent of a correlation = 1.0)

This is the regression line of receipts on theater capacity. The line actually goes through the "points of averages" (page 162) that is, for each value of theater capacity, the regression line is going through a best fitting line, a smoothed line, of the average receipts for a given level of theater capacity.

From your text, the regression line of the y-variable (receipts) on the x-variable (theater capacity) is giving an average y-value (average receipts) for each value of x. (page 160)

Freedman summarizes the method by stating that for each one standard deviation increase in X, there is a r*(standard deviation increase in y). The change in Y in response to X is not a one to one increase, it is not an increase of the correlation either, it's a combination of the correlation between X and Y and the standard deviation of y.

C. REGRESSION FOR INDIVIDUALS (10.3) -- PREDICTION

Chapter 10.3 has an easier way to construct this line and allows you to work on the prediction of individual values of Y given the value of X. Back to the example above:

A new play comes out and has a theater capacity of 6000. Predict the receipts for that play.

The play is (6000-9997.333)/2664.093 = -1.50 standard deviations below the average theater capacity (the X variable) (recognize the formula? It's Z)

If we take the –1.50 standard deviations and multiply it by the correlation .82 we can relate it to the Y variable (receipts) and get a –1.23 standard deviations for Y.

We can now give a prediction for receipts by multiplying –1.23 by the standard deviation for receipts of 187,723.80:

-1.23 * 187,723.80 = -230,900.27

and adding this result to the average total receipts (345,532.70+ -230,900.27) = $114,632.43 or so.

So if a new play has a capacity of 6000, given what we know about capacity's relationship to receipts, we predict (or expect) receipts of about 114,632.43

4. USING THE Z SCORE AND REGRESSION INFORMATION (10.3)

Notice we used a Z score above. We can do something similar to Chapter 5. Suppose a play is in the highest 10% of theater capacity and suppose theater capacity is normally distributed. Recall that if f you are in the top 10% of ANYTHING, this means you have a Z = 1.3 or you are 1.3 standard deviations above average (remember, a Z score could be thought of as just the number of standard deviations you are above or below the average, Z's are called "standard units").

If a play is in the top 10% of theater capacity, it's capacity is calculated to be 9997.333 + (1.3 * 2664.093) = 13460.654

And so, its corresponding total receipts will be will be:
(1) find the product of the Standard Deviations of capacity (X) and the correlation: 1.3* .82= 1.066
(2) multiply this result by the Standard Deviation of receipts (Y): 1.066 * 187,723.80= 200,113.57
(3) add this value to the average total receipts (Y) 200,113.57 + 345,532.70 = 545, 646.27

Therefore, if your play is in the top 10% of theater capacity, the corresponding receipts is predicted to be 545,646.27