1. The Standard Normal Distribution

Used to approximate or describe histograms of many (but not every) types of data. Properties are:

a. Symmetric, bell-shaped, the "bell curve", see page 86-87 of your textbook.

b. Mean 0, SD 1

c. The median is where 50% (half) of the observations are on either side. In this distribution, the mean is equal to the median. The values on the horizontal axis are called "Z SCORES" or "STANDARD UNITS". Values of Z above the average are positive, values of Z below the average are negative.

d. Area under the curve is equal to 100% when expressed as a percentage. The shaded area under the curve represents the percentages of the observations in your data to the left of given values of Z.

e. 68%-95%-99.7% rule (see p. 87) About 68% fall within plus or minus 1 SD of the mean About 95% fall within plus or minus 2 SD of the mean Nearly 100% (99.7%) fall within plus or minus 3 SD

f. The curve never crosses the horizontal axis, it gets very close at the extremes though. It extends to negative and positive infinity.

2. Standard (Deviation) Units

Recall that a variable's value has been converted to a score Z if the value of Z tells one how many SD's the original value is above or below the average.

$$z = \frac{(\text{value of interest} - \text{average of all the values})}{\text{standard deviation of all the values}} = \frac{y - \mu}{\sigma}$$

Z is the result of converting some value of interest (y) to a score that can be interpreted as the number of standard deviations the original unconverted value is above or below the average.

4. More Examples of the use of Standard Units

See handout on heights and weights of models. The average height of US Women over age 18 is 65 inches (about 5'5") with a standard deviation of about 3 inches. Elle McPherson is a fashion model, she is listed at 6' tall (72") using standard units we can compare her to the average US woman

$$z = \frac{(72 - 65)}{3} = \frac{7}{3} = 2.33$$

or she has a Z score of +2.33. She is 2.33 standard deviations above average in height. She is taller than .9901 or 99.01% of all adult women. See Table Z on page A-82-83 105, look up +2.3, then find the second decimal place as 0.03 to make +2.33. The .9901 is the shaded area or the total area (percentage) to the left of a Z=+2.33

What percentage of women are taller than her?

Your professor is average height at 5'5", what percentage of women are taller than your professor but shorter than Elle?

If the data are normally distributed, then raw scores can be converted into standard units to find percentages; also, percentages can be converted into standard units and then converted back into raw scores (original numbers).

5. More Converting Standard Units back to original values

Examples of the GRE (mean = 1000, SD =200). UCSB Geography Department graduate students had an average 1315 and the minimum score considered was 1100. What percentage of GRE test takers can consider applying? What percentage of GRE test takers have scores at or above 1315?

Examples

a. GMAT scores are normally distributed with a mean of 500 and an SD of 100. Total scores range from 200 to 800. UCLA's Anderson school reported an average score of 698 for the graduating class of 2002. What percentile rank is a score of 698?

(first step: draw a picture ... want an area)
(next step: convert to standard units; z = 1.98)
(final step: look up the area; 97.61 percentile)

b. What GMAT score defines the top 10%?

(first step: draw a picture ... want an original score)
(next step: using the area, find the standardized score: z=+1.28)
(final step: convert back from standard units; score is 628)

c. Stanford Business School reports a class average score of 727. What is the percentile rank of the average Stanford MBA? What percentage of all test takers fall between UCLA's typical 698 and Stanford's typical 727?

6. For next time: Please read Chapter 11, we'll talk about it briefly.

A. Randomness exists
B. Simulations model random events
C. Outcomes and Trials