1. Random Variables have variances too (pp. 305-306)

A random variable has a measure of spread: the variance and its square root, the standard deviation. The variance (and standard deviation) of a random variable measures how far the various outcomes are from the mean or expected value. Just like the definition of variance or standard deviation in Chapter 5, there is a similar interpretation -- it's the average (typical) deviation from the mean (in this case, the expected value).

Symbol: variance is σ^2 and standard deviation is σ. Generally they have a subscript like x to tell you that it is the variance of random variable x or the standard deviation of random variable x.

Notes: some textbooks call the standard deviation of a random variable the "standard error" in order to distinguish this from the standard deviation you learned about in Chapter 5. Like the mean in Chapter 5 versus the mean of a random variable in Chapter 16, this standard deviation in Chapter 16 is for a RANDOM PROCESS or a THEORETICAL VALUE. The mean and standard deviations from Chapter 5 are used for a list of numbers.

2. The variance and standard deviation of a discrete random variable (p. 335)

Just like the mean of a discrete random variable weights each outcome by their respective probabilities, the squared deviations are weighted by their probabilities. For a discrete random variable X, the variance is:

$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu_x)^2 \, p_i$$

where n is the number of possible outcomes.

The standard deviation is the square root of the variance.

Example: A salesman makes 3 calls per household over the course of one year, how many sales can he expect to make?

f(x)	0	1	2	3
p(x)	.50	.30	.15	.05

The expected value or mean was .75 sales

The variance then is:

$(0 - .75)^2*.50 + (1-.75)^2*.30 + (2-.75)^2*.15 + (3 - .75)^2*.05 = .7875$ sales squared

The standard deviation is a little easier to interpret

Square root of .7875 = .8874 so .8874 sales

Interpreting these two will be a little easier in coming days. For now, interpret them like the mean and standard deviation you are familiar with, just keep in mind that these are the means and standard deviations of random variables.

3. The variance and standard deviation of a continuous random variable

Like the mean of a continuous random variable, it will be given to you. You won't be asked to calculate the variance of a continuous random variable. Again, normally distributed random variables are the best examples of a continuous random variable. You are given a mean and a standard deviation and will be expected to make statements about the distribution.

4. Rules and properties of the mean of a random variable

Rule 1. $\mu_{a+bX} = a + b(\mu_x)$

If a is a constant and b is a constant, then the mean of random variable is the constant plus the random variable and the mean of a constant times a random variable is the constant times the random variable.

Example: The salesman has 100 homes in his region. Question: How large is his expected annual income given that he calls each home 3 times in a year and he gets $200 per sale as a commission and his base salary is $12,000 annually?

From last time, his expected value (or mean) μ_x is .75. If he has 100 homes in his region, he's expected to make 75 sales in 300 phone calls (he has 100 homes, he calls each 3 times, he's expected to make a little less than 1 sale for every 3 calls). If he gets $200 per sale that's b(μ_x)= $200*75=$15,000 if his base salary is $12,000 that's a + b(μ_x) 12,000+15,000=27,000.

Rule 2. Given random variables X and Y, $\mu_{x+y} = \mu_x + \mu_y$. You can add the means of two different random variables together.

The salesman falls in love with another salesperson who is better on the phone, that salesperson also calls each home 3 times a year and also gets $200 per sale. However, this salesperson's expected value (or mean) μ_x is 1.25. Together, they have an expected value of 1.25+.75 = 2. So for each household contacted, they can expect 2 sales.

5. **Rules and properties of the variance of a random variable (p. 307-308)**

a. **If you multiply a random variable by a constant b, the variance will be multiplied by b-squared (b^2). If you add a constant to a random variable, the variance will not be affected by addition.**

b. **If you have two independent random variables X and Y then regardless of whether you are interested in the sum of the two (X+Y) or the difference between the two (X-Y), the variances MUST be ADDED together to get the combined variance. Do not subtract variances.**

Note: You cannot add standard deviations to get a combined standard deviation, you must find the variances, add those together and then take the square root to find the combined standard deviation.

Example: Here is salesperson 1 again

f(x)	0	1	2	3
p(x)	.50	.30	.15	.05

Salesperson 2

f(x)	0	1	2	3
p(x)	.45	.10	.20	.25

The first mean is .75 and the second is 1.25
Their combined mean is .75+1.25=2.0

The first variance is $(0 - .75)^2*.50 + (1-.75)^2*.30 + (2-.75)^2*.15 + (3 - .75)^2*.05 = .7875$ sales squared

The first standard deviation is = .8874 sales

The second variance is: $(0 - 1.25)^2*.45 + (1-1.25)^2*.10 + (2-1.25)^2*.15 + (3 – 1.25)^2*.05 =$ .9469 sales squared

The second standard deviation is =.9731

The combined standard deviation is the square root of (.7875 + .9469) = 1.317 sales

The combined earnings are 2 sales per week