1. Review

Definition. PROBABILITY is the study of CHANCE: a certain random process is given (such as rolling a die or spinning a roulette wheel), and we want to know the chance of various outcomes. Probability is how likely an event is to happen.

2. Quick Review of Basic Rules

- *The chance of something is the percentage of time it is expected to happen*
- *Chances are between 0% and 100%. In general, The probability of an event occurring is given as a fraction, decimal or percentage. 0% definitely will not happen and 100% means definitely will happen. The rest are in between.*
- *The chance of something is 100% minus the opposite thing (the complement).*
- *Multiplying Probabilities: The chance that two or more things will happen equals the chance that the first will happen multiplied by the chance that the second happens and so forth given that a prior event does not forbid the occurrence of the future event.*

3. Calculating Basic Probabilities:

A basic probability is calculated by finding:

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the number of outcomes of interest
---------------------------------- = probability of an event
total possible number of outcomes
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Using this formula requires that you know how many outcomes to expect. Example, Problem 7, Exercise Set C. A coin is tossed 3 times. What is the chance of getting at least 1 head? You need to know what can possibly happen before you can estimate the chance of getting at least 1 (that means 1, 2, or 3) heads.

4. Combining Probabilities

When finding the probability of two or more events occurring one after the other, you need to find out the probability of each of the events occurring. Then you see whether you add or multiply the probabilities.

If one AND another event has to occur you MULTIPLY (Chapter 13)

If one OR another event has to occur you ADD (Chapter 14)

Being able to list the ways events can occur or using diagrams can help you calculate probabilities for combined events.

5. Listing the Outcomes or Listing the Ways (14.1)

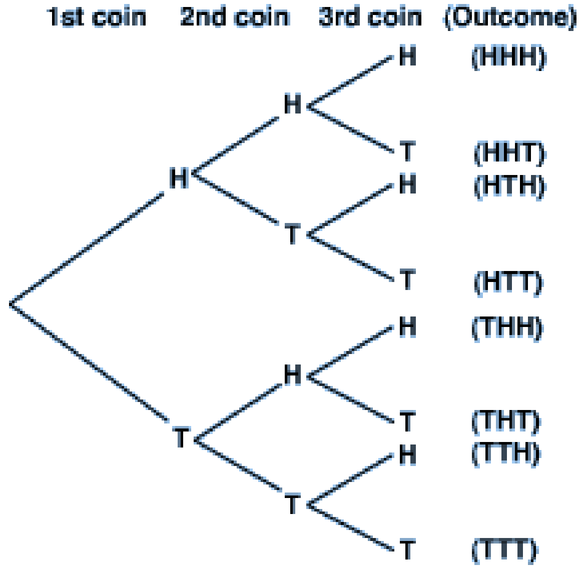
For some problems it might help you to list ways and it may help you to understand calculating chances. If you toss a fair coin, you have two possible outcomes.

The chance of the coin landing heads up is 1/2.
The chance of the coin landing tails up is 1/2.

Let's move to a situation where you are tossing a coin 3 times, there are 8 possible outcomes (2x2x2):

HHH HHT HTH HTT THH TTH THT TTT

Your book doesn't go into this but tree diagrams enable us to see all possible outcomes and to calculate probabilities. From the tree diagram we can see that there are eight possible outcomes.



Examples

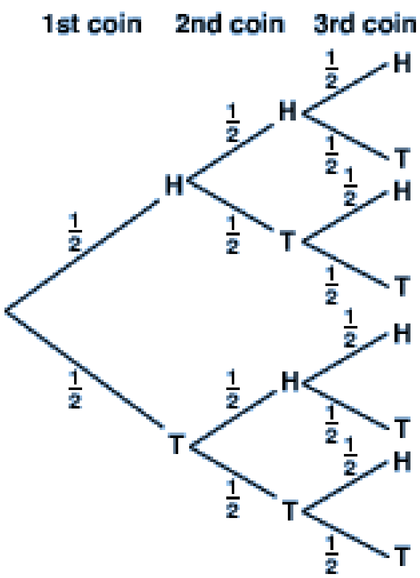
Three coins are tossed. What is the chance of:

1. *getting three heads?*
2. *getting two heads and one tail?*
3. *not getting 3 heads?*

Only one of these is three heads, so the probability of getting three heads 1/8
Three of the outcomes show two heads and a tail, so the probability of getting two heads and a tail is 3/8
Not getting 3 heads is the same as 1-(probability of getting 3 heads) = 7/8

This method is very effective when there are only a few possible outcomes and when the events are independent, but in most cases you will need to calculate the answer using probabilities.

Look at the tree diagram again, this time with the probabilities included:



$$P(HHH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(\text{two heads and a tail}) = P(HHT) + P(HTH) + P(THH)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= \frac{3}{8}$$

NOTES:
The sum of the probabilities for any set of branches should always be 1.
*We **multiply horizontally** and **add vertically**.*

Example: you have 5 employees in your store. Employee A shows up 90% of the time, B shows up 75% of the time, C shows up 50% of the time, D shows up 90% of the time, and E shows up 60% of the time. What is the chance that at least one employee will be absent on a given day? Try listing the ways....(don't)

Instead, you might think about: Chance at least one absent = 100% - (chance of all 5 not absent)

*5. **AND vs. OR rule***

*We already know that if two events are **independent**, then the outcome of one has no effect on the outcome of the other.*

*Two events are said to be **mutually exclusive** if they cannot happen at the same time. For example, if a die is thrown once, the events 'obtaining a 6' and 'obtaining a 1' are mutually exclusive as they cannot happen at the same time.*

*The events 'obtaining a six' and 'obtaining an even number' are **not** mutually exclusive as it is possible to throw a six, and this fits into both categories.*

*When two events A and B are **independent**, then:*
P(A and B) = P(A) x P(B)

*When two events A and B are **mutually exclusive**, then:*
P(A or B) = P(A) + P(B)

*Notice how the word **and** has been replaced by a **multiplication** sign for independent events, and the word **or** has been replaced by an **addition** sign for mutually exclusive events.*

*This forms the basis of the **AND vs. OR** rule for adding and multiplying probabilities*

Example
A bag contains 5 green beads and 4 red beads. A bead is taken from the bag, its color noted, and then replaced. A second bead is then taken from the bag. What is the probability that the two beads are of different colors?

The solution
*We are being asked to find **P** (1st is green **and** 2nd is red **or** 1st is red **and** 2nd is green)*
*The first bead is replaced before the second bead is taken, so the first and second beads are **independent**. Therefore the word **and** can be replaced by a multiplication sign.*

*The events '1st is green and 2nd is red' and '1st is red and 2nd is green' are **mutually exclusive**. Therefore the word **or** can be replaced by an addition sign.*
The answer to the question is

$$\frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9} = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$$

7. Summary: Adding versus Multiplying Probabilities (14.2 and 14.3)

- *Two events A and B are MUTUALLY EXCLUSIVE if the events A and B can't happen together.*
- *When asked to find the chance of at least one of two things (or more) things happening, ask if they are mutually exclusive. If they are, you can add the chances. If they ARE NOT MUTUALLY EXCLUSIVE DO NOT ADD THE CHANCES.*
- *Two events A and B are INDEPENDENT if knowing whether or not A happens does not help in predicting whether or not B happens. They can happen together, they just don't affect each other.*