1. Definition of the Expected Value (17.1)

The EXPECTED VALUE (for a sum or a count) in Chapter 17 is <u>the number of draws from a box times the average of the face value of the tickets in that box</u>. The draws must be random with replacement for this to work so the process will be unbiased and the probabilities unchanging You could associate "Expected Valu"e with the idea of a "most likely outcome"

*A basic example: a coin toss -- it has 2 outcomes. Head or Tails. Suppose we're interested in the **<u>count</u>** of heads in some number of tosses. We could assign a value of 1 if a toss comes up heads and a value of 0 if it comes up tails (because when we sum it up, it's just like a count of heads). We expect 50% for each outcome (i.e. half heads, half tails). The average of the box is .50 or 1/2 or 50% (i.e. 0 + 1 divided by 2 -- a simple average of its "tickets").*

In a situation of 10 tosses (draws), you wind up with an expected value of 5 (10 times 1/2). Or think of this situation as a count or total of 5 heads is the most likely outcome if a coin has been tossed 10 times.

More complicated examples: roulette, craps, grades, bullies…

2. The Standard Error or SE (17.2) -- Does this sound familiar?

What is suggested in 17.2 is this:

Actual Outcome (observed value) from some number of draws is = expected value + chance error

where chance error is just some amount above or below the expected value.

Think about tossing a coin ten times. If I toss it ten times and get 9 heads, you might think I'm extremely lucky or I'm cheating.

If I toss a coin ten times and get 6 heads, you probably wouldn't think I was extremely lucky or that I was cheating. 6 seems reasonable, 9 doesn't. This is where the chance error component enters. What you are sensing, intuitively, is the size of the chance error in a coin toss. For a coin box, the SD of the box also happens to be 0.5.

The standard error (SE) is an estimate of the chance error. An outcome (e.g. sum) from some number of draws will be around an expected value but it can (and will be) off by chance error.

Formula: standard error of a sum or count = $\sqrt{number\_of\_draws}$ ** (Standard Deviation of the "box")*

So for a coin box, in 10 tosses, the standard error of the sum or count = $\sqrt{10}$ ** 0.5 = 1.6 (approximately)*

Remember, Standard Deviation is a measure of spread. What the formula suggests is that the more draws you make, the larger the standard error for a sum or a count . Example: 4 draws, the multiplier is 2 (root 4), 9 draws it is 3 (root 9), 25 draws it is 5 (root 25), 36 draws it is 6, and so forth.

***Note**: the standard error is not the same as the standard deviation. The SD is calculated for lists, but the SE is for some kind of chance or random process, like a lottery, like drawing tickets from a box, lik e tossing a coin. The SD is part of an SE though (you need to know the SD to calculate and SE).*

3. Using the Normal Curve with Expected Values and Standard Errors (17.3)

This section ties it all together. You can borrow the normal curve to make statements about random processes (such as draws from a box, coin tosses, craps, the number of kids expected to be bullied, whatever).

All that is required is that you:

(a) *calculate the expected value of the box and*

(b) *calculate the Standard Error based on both the number of draws and the SD (standard deviation) of the box*

Then, you can calculate standard units with a familiar formula that has been modified:

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Z = (observed value - expected value)
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              Standard Error
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Example. *Let's go back to the 9 heads in 10 tosses of a coin idea. More formally, the Standard Error for the coin toss situation is* $\sqrt{10}$ * $[(1\text{-}0)*\sqrt{((.5*.5)}\,]$ *(read 17.5 for the specifics on calculating the standard deviation for a 1,0 situation). Let's see how likely it is to get 9 heads in 10 tosses.*

SE = 1.5811 and Z = ((9 -5) / 1.5811) = 2.53 or about 2.55. The area between + and - 2.55 is 98.92% which leaves 1% total outside of the area. So the chance of getting 9 heads or better is about 1/2 of a percent. The chance of getting 6 heads or more is about 25%

Your intuitive sense works well. The combination of the expected value, standard error, and normal curve validates your suspicions.

This same method can be used to figure out chances in all kinds of situations.

4. SHORT CUTS & COUNTING (17.4 and 17.5)

Finding Standard Deviations in this chapter can be difficult. Freedman offers you some handy short cuts

(a) *If you have a situation with only two numbers, a quick formula for the standard deviation is:* *(big number - small number)* * $\sqrt{\text{fraction with big number} - \text{fraction with small number}}$

(b) *If you have a situation with only two numbers and you can make one of them a zero and the other a one, then the formula becomes:* $\sqrt{\text{fraction with big number} - \text{fraction with small number}}$

(c) *In all other situations, you will need to "expand" the box by converting fractions into real relative values sometimes, for example, suppose you have 3 numbers in a box in the following proportions:*

-1	0	+1
1/6	1/2	1/3

This is the same as: -1, 0, 0, 0, 1, 1 and the average is .16667 and the S.D. is .6872. So if you had 12 draws from this box, you would expect a sum of 2 and a standard error of $\sqrt{12}$ * .6872 or 2.38