Statistics 10 Lecture 16 The Accuracy of Percentages (Chapter 21.1-21.3)

1. Overview

In Chapters 17-20, the parameters were known, many samples are taken, and then we estimate the chance of a specific outcome. In Chapter 21 we begin STATISTICAL INFERENCE, here the parameters are unknown, and we draw conclusions from sample outcomes to make guesses about the value of the parameters. We are given a single sample and then ask the question -- what did the "box" that generated the sample outcome look like?

In this chapter we will examine CONFIDENCE INTERVALS (21.2) for estimating the value of the population parameter. The confidence intervals are based on the sampling distribution of statistics from Chapter 20.1 An important point to remember, population parameters, although unknown, are fixed (they do not change). It was the OUTCOME (statistic from a sample) that was random. Randomness, that is either your data comes from a random sample or from a randomized experiment, is an important prerequisite.

2. A Real Life Example

Up until now, we have complete information on the "box" – that is, we know what the outcomes or tickets are and we know what frequencies or percentages they have. But in reality, we almost never have complete information. So, statisticians **substitute statistics from the sample** and perform something like normal calculations as if they actually had parameter values. In other words, statistics substitute for parameters with a disclaimer attached. Here is an typical example. A poll was conducted on April 20-24 in advance of Timothy McVeigh's execution. It shows that majority of Americans support the death penalty (63%). A poll on gas prices says that 48% of Americans say that rising prices have caused financial hardship in their homes. Note: in each of these examples, the" fine print"says the margin of error is three percentage points. A third poll from the local paper says that 63% of Californians think energy is California's #1 problem. At the end of this poll, it says the survey had a margin of sampling error of + or – 4 (percentage) points.

3. What is "the margin of error"?

Remember this from the notes on Chapter 17:

Observed Outcome = Expected Outcome + Chance Error (also see p. 381)

The Expected Outcome is the Expected value and the Chance Error is estimated by the Standard Error.

What the two polls are reporting are OBSERVED OUTCOMES or statistics from a sample. The margin of error relates to the Standard Error. We do not know what the expected outcome is (we would know if we had the parameter value or knew exactly what the box looked like, but we don't) but we do have an observed outcome to work with and some estimate of chance error.

"Margin of Error" as it is called in the media and "Confidence Interval" as it is called your book (21.2) re closely related and they are both indicators of the "strength"/"believability"/"accuracy" of a statistic (e.g. 63% support the death penalty, 48% say gas prices cause hardship, 63% say energy is the #1 problem) obtained from a sample. They are BOTH expressions of confidence in what conclusions you might draw from a survey result.

A "margin of error" is the same as a confidence interval with a confidence level of 95%. We will spend today and next lecture talking about what that means and how to calculate it and how to interpret it.

4. Calculating the "Margin of Error" for a sample percentage

From the poll on the death peanlty, let's treat "favor" as a "1" all other responses as a "0". Let's call these the tickets and put them in the 'box".

Then let's substitute our percentages from the sample for our box percentages (which are actually unknown).

We can now calculate a box SD ((1-0) $\sqrt{.63 * .37}$ = .4828
and then from the box SE we can calculate the SE of a percentage. Since there are 1,003 who were surveyed, it is:

$\frac{\sqrt{1003} * [(1-0) \cdot \sqrt{.63 * .37}]}{1003} * 100$ = 1.52% (notice that this $\frac{SE_{number}}{samplesize} * 100$ is from p. 360)

They rounded it to about 1.5 percentage points. And then multiplied the 1.5 percent by 2 (so we have 2 S.E.s here) and then reported "+ or - 3 percentage points". What are they doing?

5. Confidence Interval Basics (21.2)

A CONFIDENCE INTERVAL then is a range of values (i.e. values derived from sample information) that we think covers or contains the true parameter. Reuters reported that the "margin of error" for death penalty support was about 3% for adult Americans. This suggests a range around the sample statistic of 60% to 66% as the current national level of death penalty support. This interval is supposed to cover or contain America's true support. This is about plus or minus 2 Standard Errors and is the way the media expresses results from polls. What they are saying is that they were "95% confident that the interval 60% to 66% covers the true percentage of American support for the death penalty".

In our other examples then, we are "95% confident that the interval 45% to 51% covers the true percentage of American reporting hardship because of rising gas prices" and for the last one? We are "95% confident that the interval 59% to 67% covers the true percentage of Californians reporting energy prices as the #1 problem"

The figures 48% plus or minus 3% (etc) are confidence intervals for the population percentage and they are calculated from sample percentages and sample standard deviations. Up until now, we've been in a situation where we know exactly what the "box" looks like, now we don't, but we have samples which can reveal "the truth" (i.e. the parameter).

6. SO WHERE DO YOU THINK THEY GOT THE 95% FROM? WHERE HAVE YOU HEARD IT BEFORE?

From the normal! Review pages 355-359 if this is unclear. If we could sample and resample using samples of the same size, in the long run (or infinitely) , and if we plotted the outcomes, we would see a normal curve arise. Our statistics from our one sample is one possible outcome – but it is somewhere on the normal curve. We are just uncertain as to where exactly…but we can be 95% confident that the range we gave (based our our statistic) contains the true percentage (the number we really want).

7. Properties of Confidence Intervals

In about 68% of all samples, the sample percentage will be within one standard error of the population percentage. So from the poll outcome, we would say that we were 68% confident that between 61.5% and 64.5% support the death penalty.

In about 95% of all samples, the sample percentage will be within two standard errors of the population percentage.

From the poll, we would say that we were 95% confident (this is also the standard margin of error reported in the media) that the percentage of support is 60% to 66%

In about 99% of all samples, the sample percentage will be within three standard errors of the population percentage. From the poll, we would say that we were 99% confident American support for the death penalty is between 58.5% and 67.5%

You can never been 100% confident. There is always the chance that you could have a very bad sample and are nowhere near the true population parameter.

Please note that the parameter is fixed and unchanging. Our sample statistics will change from sample to sample. See the figure on page 385 in your text. If 80 is the parameter, the lines represent confidence intervals for 100 different samples. Notice that a few never "cross" the line.