1. A pumpkin farmer produces pumpkins that average 12lbs in weight, with a standard deviation of two pounds. The pumpkins are packed 16 to a crate. What is the chance that a single crate weighs more than 195 pounds?

(a) Less than 1%
(b) About 7%
(c) About 13%
(d) About 34%
(e) About 69%

$16 \times 12 = 192$ lbs

$$z = \frac{195-192}{\sqrt{16} \cdot 2} = \frac{3}{8} \approx +.40$$

The next 5 problems are related to each other and use the information below:

There are 20,000 restaurants in the County of Los Angeles, 50% of them received a letter grade of "A" during inspections, 40% received either a B or a C grade and 10% failed their inspections. Restaurant grades are not normally distributed. My financial adviser, the Oracle, has hired you as a temporary personal assistant. Your job is to schedule his next 16 dinners (Oracle never eats at home). Unfortunately, you didn't know about the rating system and you never eat out because you don't have the money. So you listened to your best friend and picked 16 restaurants at random from an internet database of the 20,000 restaurants in Los Angeles. The Oracle will give you +3 points if you choose "A" restaurants, +1.25 points if you choose "B" or "C" restaurants, and -20 points if you choose a restaurant with a failing grade. Treat your restaurant selections as if they were a simple random sample of restaurants.

2. Construct a box model for this problem

$\boxed{+3}$ $\boxed{+1.25}$ $\boxed{-20}$

.50 .40 .10

Box Average = 0

3. What is the expected value for the total score of the 16 restaurants selected at random?

$$EV_{sum} = 16 * ((+3 \times .5) + (+1.25 \times .40) + (-20 \times .10))$$

$$= 0$$

4. What is the standard deviation for the box?

$$SD_{BOX} = \sqrt{(.50)(+3-0)^2 + (.40)(+1.25-0)^2 + (.10)(-20-0)^2}$$

$$= \sqrt{45.125}$$

$$= 6.7175$$

5. What is the standard error for the total score of a sample of 16 restaurants?

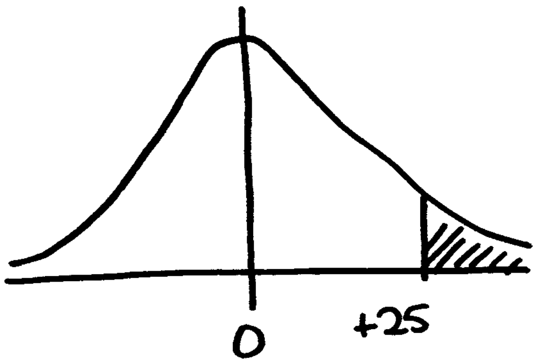
$$SE_{sum} = \sqrt{16} \cdot 6.7175$$

$$= 26.8701$$

6. To convert your temporary job into a permanent job, you must have accumulated a total of at least +25 points from the Oracle after picking 16 restaurants for him. What's your chance of getting a total of at least +25 points after picking 16 restaurants? If it is not possible to calculate the chance, please write "not possible" below and explain why.

The best answer here is to argue that the normal approximation does not apply because 16 draws is NOT REASONABLY LARGE (see Ch. 18 parts 5+6) (and Ch. 18 summary point 3) – therefore it is not possible.

But you'd probably get some credit for doing the work so here is a solution:



$$z = \frac{25-0}{\sqrt{16} \cdot 6.7175} = .93 \approx {}^{+}.95$$

area between $-.95$ and $^{+}.95$ is 65.79%

so chance is $\dfrac{100-65.79}{2} = 17.105 \approx \boxed{17.1\%}$

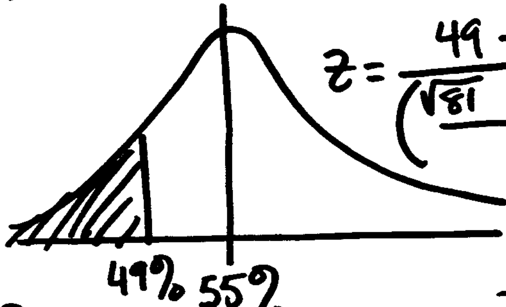
7. Suppose we are psychics and we know that former police chief Bernard Parks will be the next Mayor of Los Angeles with a final winning percentage of 55%. Unfortunately, we don't know Parks and he doesn't return our phone calls or e-mails so he doesn't know he will get 55% of the vote after the next election. In fact, he is spending a lot of money right now on random surveys of votes of size 81 to help him make decisions about the upcoming election.

a. What is the chance that one of his surveys will give a result showing that he will get 49% or less of the vote if it is true that he really has 55%?

Box is KNOWN

for Parks → [1] [0] ← against

.55 .45



$$z = \frac{49 - 55}{\left(\frac{\sqrt{81} \cdot \sqrt{.55 \times .45}}{81} \times 100\right)} = -1.09$$

$z \approx -1.10$

area between ± 1.10 is 72.87% so tail is $\boxed{13.565\%}$

Some bad question → b) Suppose again that Parks does not know that he will get 55% of the vote and suppose he takes another random survey of size 81 and it shows that 49% will vote for him. Can ~~you~~ he construct a 95% confidence interval for the population percentage of votes for Parks?

(should give you a level of confidence)

(should have read he since you know the parameter)

Circle one: Yes No

If you circled yes, please construct a 95% confidence interval. If you circled no, please explain why ~~you~~ he cannot construct a confidence interval.

Sure, use his sample information

$$49\% \pm 2 * \left(\frac{\sqrt{81} \cdot \sqrt{.49 \times .51}}{81} \times 100\right)$$

$$\pm 2 * (5.5544)$$

$$\boxed{49\% \pm 11.11\%}$$

8. There were a total of 226,324 deaths in California in 1999. A random sample of 242 deaths was selected. Detailed research determined that the deceased was cremated in 99 of the deaths.

a. Determine a 99% confidence interval for the proportion (or percentage) of deaths in California in which the deceased is cremated.

$\frac{99}{242} = .4091$ $\approx 41\%$

$41\% \pm (2.6)\left(\frac{\sqrt{242} \cdot \sqrt{.41 \times .59}}{242} \times 100\right) = 41\% \pm 8.22\%$

use $z = 2.6$ for 99%
(using 3 is OK)

b. Suppose the confidence interval is too narrow, identify 2 things you can do to make the interval wider.

1) increase confidence
2) decrease sample size

c. A classmate comes up to you and says, this is the interpretation of a 99% confidence interval:

"There is a 99% probability that the true parameter is in the interval you gave in part (a)"

Is your classmate's interpretation correct? (circle one) YES NO
And justify your choice in the space below.

It is wrong to talk about the true parameter having a probability.
A parameter is a fixed value.

Instead it is the statistic and the endpoints of the confidence interval which have probabilities (or chances)
99% refers to the percentage of intervals over the long run that contain the parameter.
You have a 99% chance of picking a good interval.

9. You know that every UCLA student will definitely get a job after graduation. The only uncertainty is the salary. Suppose this is what you know about the job prospects of UCLA students after graduation:

There is a 35% chance that the salary will be \$20,000 per year; a 45% chance that it will be \$90,000 per year; and a 20% chance that it will be \$40,000 per year. Suppose you draw a random sample of 9 UCLA students.

a. Draw a reasonable box model for this problem

20	90	40
.35	.45	.20

← in thousands

Box Average is 55,500

b. Find the expected value of the total (sum) salary for the 9 UCLA students.

$$EV_{SUM} = 9 \times 55{,}500 = \boxed{499{,}500}$$

in thousands

c. What is the Standard Deviation of the "box" you drew?

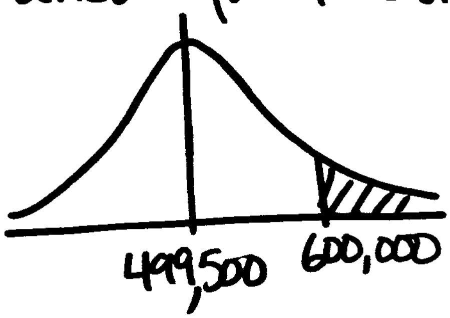
$$SD_{BOX} = \sqrt{.35(20-55.5)^2 + .45(90-55.5)^2 + .20(40-55.5)^2}$$

$$\boxed{= \$32{,}012}$$

d. What is the Standard Error of the total (sum) salary for 9 students?

$$SE_{SUM} = \sqrt{9} \cdot 32{,}012 \approx \boxed{\$96{,}036}$$

e. Suppose you work for me and I tell you to go draw a different random sample of 9 UCLA students and you get a total (sum) salary of \$600,000. What is the chance that you could have gotten a total salary this large or larger? Yes, 9 is too small, but go ahead for this one, I didn't ask.



$$\frac{600{,}000 - 499{,}500}{96{,}036} = z = +1.05$$

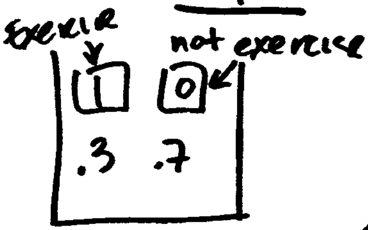
$$\frac{100 - 70.63}{2} = \boxed{14.69\%}$$

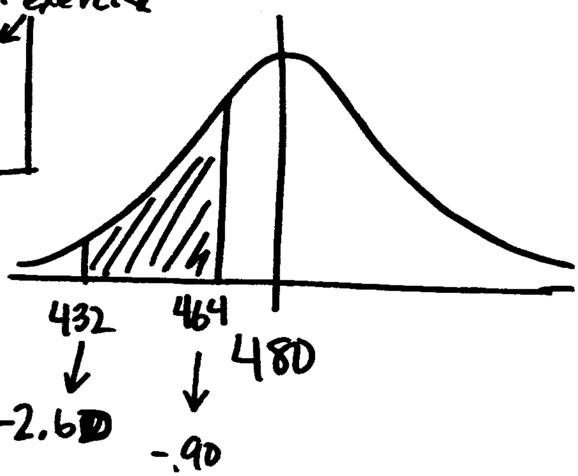
10.Exercise is particularly beneficial for young adults. A study wants examines the exercise habits of college students. A sample of 36 college students is selected by a random process and it is found that a 68% confidence interval for the percentage who exercise weekly is 22.4% to 37.6%. Using this information, please answer the following true/false questions (4 points each)

	True	False	Statement
A		X	There is a 68% chance that for all college students, percentage who exercise weekly is between 22.4% and 37.6%
B	X		If we quadrupled the sample size, we would expect the 68% confidence interval to be one half as wide.
C	X		If we increased the confidence level to 99%, we would expect the confidence interval get wider.
D		X	This 68% confidence interval is valid only when the population variable (percentage who exercise) is normally distributed.
E	X		The sample statistic that generated this confidence interval is 30%, in other words 30% of the sample was found to exercise weekly.

11. Suppose it is know than 30% of all college students exercise weekly. A physiology class needs enough students for a medically valid study and a sample of size 36 is not large enough. So if 1600 college students are chosen at random instead of 36, what is the chance that between 432 and 464 of those chosen do indeed exercise weekly? (8 points)

If 1600 are drawn and 30% exercise, we expect 480 to be exercisers so.





Need 2 z scores

$$z_{464} = \frac{464-480}{\sqrt{1600}\cdot\sqrt{.3\times.7}}$$

$$z_{464} = -.87 \approx -.90$$

$$z_{432} = \frac{432-480}{\sqrt{1600}\cdot\sqrt{.3\times.7}}$$

$$z_{432} = -2.62 \approx -2.60$$

Area between

$z = \pm.90$ is 63.19%

$z = \pm 2.60$ is 99.07%

so $\frac{99.07-63.19}{2} = \boxed{17.94\%}$

12.A study was conducted on the sleep patterns of infants in the United States. A sample of 25 infants was drawn at random with 54% sleeping at least 12 hours a night.

a. Please compute a 95% confidence interval for the percentage of all U.S. infants who sleep at least 12 hours per night.

$$54\% \pm 2\left(\frac{\sqrt{25}\cdot\sqrt{.54\times.46}}{25}\times 100\right)$$

$$54\% \pm 19.9\%$$

b. Suppose the sample size was increased to 100 infants, what effect would this have on the confidence interval? Assume that 54% of them slept at least 12 hours per night.

it would decrease the width (divide in $\frac{1}{2}$)

or $54\% \pm 9.97\%$

c. Please calculate a 90% confidence interval for the percentage of all U.S. Infants who sleep at least 12 hours per night. Again, please use the sample of size 25 and assume the percentage in the sample was 54% sleep at least 12 hours per night.

for 90% use $z = 1.65$

$$54\% \pm 1.65\times\left(\frac{\sqrt{25}\cdot\sqrt{.54\times.46}}{25}\times 100\right)$$

$$\boxed{54\% \pm 16.4\%}$$

13. It's a family tradition: your professor goes to Las Vegas every year for Thanksgiving. A new casino has opened and they are playing a modified roulette game that has 40 possible numbers that can be spun on a wheel. To play you bet \$4 and you get to choose 4 numbers. If the wheel lands on any number that you chose, you win \$10. If the wheel does not land on a number you choose but on one of 8 "special numbers" you don't win or lose anything. If the wheel lands on any of the remaining numbers (not the ones you choose or the special numbers), you lose your bet of \$4. Suppose the typical person plays 25 times.

a. This game of modified roulette can be represented by a box model, please construct a reasonable one in the space below.

+10	0	-4
.1	.2	.7

.1 because $4/40 = .1$

.2 because $8/40 = .2$

and $1.0 - .2 - .1 = .7$

or $28/40 = .70$

b. The 25 plays can be treated like a random sample of size 25. Find the expected value of this game.

$EV = 25 \times -1.8 = -45$

(box average is $(10 \times .1) + (0 \times .2) + (-4 \times .7)$)

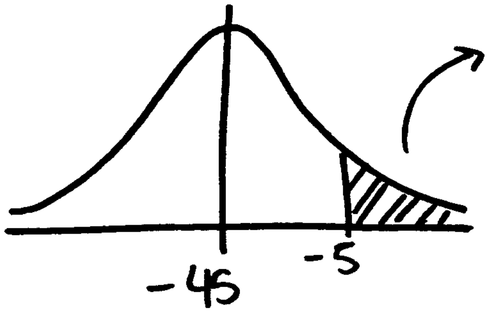
c. Find the standard error of this same game.

$SD_{BOX} = \sqrt{(.1)(10 - -1.8)^2 + (.2)(0 - -1.8)^2 + (.7)(-4 - -1.8)^2}$

$SE = \sqrt{25} * 4.2379$

$SE = 21.1896$

d. Suppose the professor decides to spend \$100 playing a total of 25 times and she lost \$5. Calculate the chance that your professor could lose 5 or fewer dollars playing this game. Show all of your work and answer this question – based on your calculations is she lucky? (let's suppose lucky means the chance of losing 5 or fewer dollars is less than 5%)?



LUCKY

find z

$z = \frac{-5-(-45)}{21.1896} \approx +1.90$

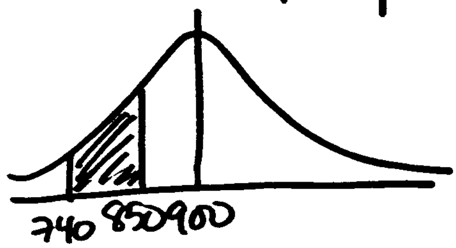
area for ± 1.90 is 94.26

so $\frac{100 - 94.26}{2} = \boxed{2.87\%}$

14. A survey of Los Angeles is conducted and 3,000 households are drawn at random. Suppose 30% of all households in Los Angeles have only one person living there. What is the chance that the number of households with only one person will be in the range 740 to 850?

bad problem

if 30% have only one person we expect 900 households in a sample of 3000. (see #11 for a similar problem)



$$z_{850} = \frac{850-900}{\sqrt{3000}\cdot\sqrt{.3 \times .7}} = -1.99$$

$\boxed{\text{ABOUT } 2.3\%}$

$$z_{740} = \frac{740-900}{\sqrt{3000}\cdot\sqrt{.3 \times .7}} = -6.37$$ ← this is off the chart

15. A multiple-choice quiz has 25 questions. Each question has 5 possible answers, one of which is correct. Four points are given for each correct answer, but a point is taken off for a wrong answer. A passing score is 30.

a. If a student answers all the questions at random, what is the chance of passing?

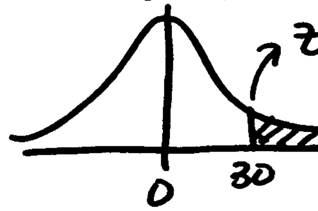
+4	-1
.2	.8

Box Average = 0 so EV = 0

Box SD = $(4 - -1)\cdot\sqrt{.2 \times .8} = 2$

so SE = $\sqrt{25} \times 2 = 10$

$z = \frac{30-0}{10} = +3$ area between $z = \pm 3.0$ is 99.73 so $\frac{100-99.73}{2} = \boxed{.135\%}$



z = +3.0 so chance is

b. Could a student score 10 points or more just by guessing? Answer yes or no and explain.

Yes. If we use the information above

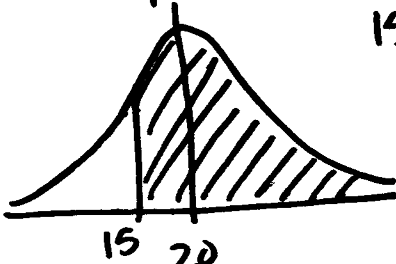
$z = \frac{10-0}{10} = +1$ and the chance is estimated to be about 16%

16. From their extensive records, the IRS knows that 20% of federal income tax returns have an arithmetic error serious enough to warrant an audit. If 100 tax returns are randomly selected, what is the probability that at least 15 of them have arithmetic errors?

20% of 100 means they expect 20

15 or more is shaded

use z



$$z = \frac{15-20}{\sqrt{100}\cdot\sqrt{.2 \times .8}} = \frac{-5}{4} = -1.25$$

↓

$z = -1.25$

area is 78.87 in the middle and 10.565 in ~~each~~ one tail so 78.87 + 10.565 = $\boxed{89.435\%}$