1. Measuring "Spread"

In addition to knowing the center of a distribution of data, it is important to know the "spread" or how far from the center data tends to be.

2. The Standard Deviation (SD)

The Standard Deviation may be thought of as the average size deviations of the individual values of a list from the overall average(the mean) of the list. So it's based on the mean.

a. In general, most numbers in a list are only one standard deviation away from the average. A few numbers will be two standard deviations away. And very few will deviate beyond that. STANDARD DEVIATION is abbreviated as SD or as a lowercase "s".

b. The SD is defined as follows: given a list of n numbers $x_1, x_2, \ldots, x_n$,

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$$ or simplified $$s = \sqrt{\frac{\sum_1^n x_i^2 - \left[\frac{(\sum_1^n x_i)^2}{n}\right]}{n}}$$

3. An Example

An interview with 5 UCLA students reveals the time in hours spent surfing the web in a given week: 7, 17, 3, 14, 7

Value	7	17	3	14	7	Sum = 48	Mean =48/5 = 9.6
Deviation from Mean	-2.6	+7.4	-6.6	+4.4	-2.6	Sum = 0	
Squared Deviation from Mean	6.76	54.76	43.56	19.36	6.76	Sum=131.2	SD= $\sqrt{131.2/5}$ = 5.1225

4. More Properties of the Standard Deviation

a . Note that the standard deviation is in the same units as the data. This is why the standard deviation involves the square root, it allows for easier interpretation. It is always a positive value.

b. As with the mean, it is not necessary to know HOW MANY items are in a list when computing a standard deviation, only the relative frequency of the values in the list.

c. The SD measures how close the numbers in the list are to the average; i.e., not all numbers are equal to the mean; the SD is a measure of the "average" distance between each point and the overall average.

d. Another thing about the standard deviation. For many datasets 68% of the entries on a list will fall within one SD of the average. 95% of the entries will fall within two SD of the average. But we'll talk about this some more in Chapter 5.

Example 1. Which one is the less riskier investment and why?

Mutual Fund (as of 12/31/02)	Average Annual Return since 1997	Standard Deviation
Wadell & Reed Science and Tech (WSTCX)	13.91%	44.84%
PIMCO Real Return Bond (PRTNX)	9.22%	3.9%

PIMCO is more consistent, we can see that by comparing their standard deviations.

Example 2. Here are some statistics on Los Angeles in 1990 and 2000. What trend can we discern from this information?

Income	Mean	Standard Deviation
Family Income in 2000	56,782.30	29,597.02
Family Income in 1990	42,543.12	22,175.07

Example 3. Coached and Uncoached SATs.

Effects of Coaching on SAT I Scores

		Pre-test Mean Score	Post-test Mean Score	Gain	Coaching Effect
Verbal	Coached	500 (92)	529 (97)	29 (59)	29 – 21 = 8
	Uncoached	506 (101)	527 (101)	21 (52)	
Math	Coached	521 (100)	561 (100)	40 (58)	40 – 22 = 18
	Uncoached	505 (101)	527 (101)	22 (50)	

Note: Numbers in parentheses are standard deviations.

Points to Note

- *Coaching companies' current estimates of the benefits of coaching for the SAT I are much too high. Coached students are only slightly more likely to have large score gains than uncoached students. In addition, about 1/3 of students experience no score gain or score loss following coaching.*
- *The typical gain associated with coaching is 8 points for verbal and 18 points for math. Coaching seems to result in about one more verbal question correct for 25–30 hours of effort and one more math question correct for 8 or more hours of effort.*
- *Results are based on a survey of 1995–96 SAT I test takers of whom 427 were coached and 2,733 were uncoached. For purposes of this study, coaching is considered to include any and all activities conducted in special preparation programs offered to students outside their schools.*

Source References:

"Preparing for the SAT — An Update" by D.E. Powers, College Board Report No. 98–5, College Entrance Examination Board, New York: 1998, ETS Research Report No. 98–34, Educational Testing Service, Princeton, NJ: 1998; also see "Effects of Coaching on SAT I: Reasoning Test Scores" by D.E. Powers and D.A. Rock, College Board Report No. 98–6; ETS Research Report No. 98–35, College Entrance Examination Board, New York: 1998; "Effects of Coaching on SAT I: Reasoning Test Scores" by D.E. Powers and D.A. Rock, Journal of Educational Measurement, 36 (1999), 93–117.

PRINCIPLES: (1) It's not enough to know the mean or average of a variable, it is necessary to know something about spread (standard deviation) (2) the standard deviation by itself is not particularly interesting, it's more useful when viewed alongside a mean or in comparison with other standard deviations.