Example. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A sample of 25 persons is drawn. How likely is it to get a sample average of 108 or more? (0.38%) How likely is it to for the first score to be 108 or more? (29.8%)

$$Z = \frac{\bar{y} - \mu_Y}{\frac{\sigma_Y}{\sqrt{n}}} = \frac{108 - 100}{\frac{15}{\sqrt{25}}} = +2.67$$

$$Z = \frac{\bar{y} - \mu_Y}{\frac{\sigma_Y}{\sqrt{n}}} = \frac{108 - 100}{\frac{15}{\sqrt{1}}} = +.53$$

Example: The Recall of Governor Davis in October 2003, Final Count (population) Treat the YES=1 and the NO=0

```
     Davis |      Freq.      Percent          Cum.
-----------+-----------------------------------------
        NO |   3,559,436        44.63          44.63
       YES |   4,415,398        55.37         100.00
-----------+-----------------------------------------
     Total |   7,974,834       100.00
```

```
                                Davis
-------------------------------------------------------------------
      Percentiles        Smallest
  1%          0               0
  5%          0               0
 10%          0               0          Obs             7974834
 25%          0               0          Sum of Wgt.     7974834

 50%          1                          Mean            .5536664
                          Largest        Std. Dev.       .4971116
 75%          1               1
 90%          1               1          Variance        .2471199
 95%          1               1          Skewness       -.2159131
 99%          1               1          Kurtosis       1.046618
```

The chance of getting a sample proportion (p-hat) as low as 51% or lower from a single sample of 100 is:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.51 - .5537}{\sqrt{\frac{.5537(.4463)}{100}}} = \frac{-.0437}{\sqrt{\frac{.2471}{100}}} \approx -.88$$

We use a Z score because our one sample comes from a much larger sampling distribution which is normal. The area to the left of -.88 is .1894 or they had a 18.94% chance of getting a sample proportion as low as or lower than 51% (when the true proportion was 55.37%)

Formulas for sample means and sample proportions

To use the normal calculation (Z score) for sample means, you should

1. have a sample size of at least 50 and
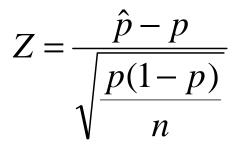2. the sample size should not be larger than 5% of the population.

The formula is then:

$$Z = \frac{\overline{y} - \mu_Y}{\dfrac{\sigma_Y}{\sqrt{n}}}$$

To use the normal calculation (Z score) for sample proportions, you should meet the following conditions

1. np > 10 & nq > 10

2. The sample size can't be larger than 10% of the population

$$Z = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$$

In either case, the samples should be random for this to work.