1. A little history

Gauss – “the law of errors”

Quetelet – applying “the law of errors” to human populations and changes it to the “normal” curve

2. The Standard Normal Distribution – An Ideal Model (table A 82-83 handout)

Used to approximate or describe histograms of many (but not every) types of data. Properties are:

a. Symmetric, bell-shaped, the "bell curve", see page 86-87 of your textbook.

b. Mean 0, SD 1

c. The median is where 50% (half) of the observations are on either side. In this distribution, the mean is equal to the median. The values on the horizontal axis are called "Z SCORES" or "STANDARD UNITS". Values of Z above the average are positive, values of Z below the average are negative.

d. Area under the curve is equal to 100% when expressed as a percentage. The shaded area under the curve represents the percentages of the observations in your data to the left of given values of Z.

e. 68%-95%-99.7% rule (see p. 87) About 68% fall within plus or minus 1 SD of the mean About 95% fall within plus or minus 2 SD of the mean Nearly 100% (99.7%) fall within plus or minus 3 SD

f. The curve never crosses the horizontal axis, it gets very close at the extremes though. It extends to negative and positive infinity.

3. Standard (Deviation) Units – The Normal as a “ruler”

A score z is in STANDARD UNITS if tells how many standard deviations an original value is above or below the average. For example, if z=1.3, then the original value was 1.3 standard deviations above average; if z = -0.55, then the original score was 0.55 standard deviations BELOW average. The formula for converting data from original units to Z scores is:

$$\text{z} = \frac{(\text{value of interest} - \text{average of all the values})}{\text{standard deviation of all the values}} = \frac{y-\mu}{\sigma} \text{ \textit{(page 86)}}$$

you can call this a "normal calculation"

4. Examples of the use of Standard (DEVIATION) Units

Law School Admissions Test scores are normally distributed with a mean of 150 and standard deviation of 10. It’s range is 120-180. When last reported, the typical law student at Yale (the #1 law school) had an LSAT score of 171. We could express that in Z scores to give us a sense of “how high”

$$z = \frac{(171-150)}{10} = \frac{21}{10} = 2.10$$

or the typical Yale law student has a Z score of 2.10. This student is 2.10 standard deviations above average in their LSAT score. He or she scored higher than or 98.21% of all LSAT test takers. See Appendix E page A-83, look up 2.1, go to the first column (labeled “0.00”) and focus on the 98.21. The 98.21 is the shaded area or the total area (percentage) <u>to the left</u> of value 2.10.

Regular consideration for admission to law school is usually an LSAT of about 141 or higher at most schools. A student with a 141

$$z = \frac{(141 - 150)}{10} = \frac{-9}{10} = -0.90$$

and the z score is associated with the number .1841 from the table. We note a few things

- This student scored higher than 18.41% of all LSAT test takers.
- This person is 9/10ths of a standard deviation BELOW average.
- The 30 point difference is 3 standard deviations (a standard deviation is 10 points in this context) it is the same as the difference in the two Z scores 2.10 - -.90 = 3.0. Remember a Z score is a standard (deviation) unit.
- There are 98.21%-18.41% = 79.8% of all LSAT test takers are between the two at this particular section of the normal curve.

5. Converting Standard Units back to original values

Idea: suppose you are told that the minimum LSAT score necessary for making the "first cut" at the UCLA school of law is Z=-1.15, what is the actual score?

-1.15 = (actual score - 150)/10 solving for the actual score you get about 138.5 or 139 on the LSAT.

Then suppose you found out that in reality, the typical student admitted to the UCLA School of Law has an LSAT score which is 2.5 Z scores above the "first cut" minimum. What's the LSAT score of the typical admitted student?

There are a couple of ways to do this, the fast way

-1.15Z + 2.5Z = 1.35Z

then 1.35 = (actual score - 150)/10 solving for the actual score you get about 163.5 or 164

Another way, one you figure out that a –1.15Z is the same as about 138.5 or 135, then add 2.5*10 (remember a Z score just tells you how many standard deviations a score is above or below average). You get the same answer.

6. Why bother with Standard Units?

Standard Units allow quick comparisons across variables with different units of measure. For example, suppose all the test scores in a class are normally distributed. The first test was worth 45 points, the mean was 33, the standard deviation was 5. A student received a 40 on that test and was told it was an A-, her Z score was

Z= (40 - 33) /5 = +1.40

The next test had a mean of 58 and standard deviation of 8. If she were to do as well on the second test as she did on the first (that is get an A-), what would her new score need to be?

Z= 1.40 = (new score – 58)/8

or she would need to score a 69.2 or something around a 69 to 70. The thing to remember is this: the second test has more points, a different mean, and a different standard deviation – it's different, but if we convert the "raw" scores to standard scores (Z), comparisons are easy. In a way, it lets us compare apples and oranges.

Try this at home, suppose you took the LSAT but decided not to go to Law School and instead applied to Graduate school but didn't take the GRE (Graduate Record Examination). Your LSAT score was 169, if you were to do as well on the GRE as you did on the LSAT, what would your GRE score be? Suppose GRE scores are normally distributed with a mean of 1000 and a standard deviation of 200.

The first thing you would need to do is convert your LSAT score in a Z score, so that's a

$$Z= (169 - 150) /10 = 1.90$$

And then translate the Z=1.90 into a GRE score or:

$$Z= 1.90 = (\text{new score} – 1000)/200$$

A 1380. Not bad.

Again, there are a couple of ways to do this, you could say

1.90*200 = 380 and add that to 1000, you still get 1380.

7. Converting the percentiles under the normal curve back to original scores

Suppose a state has a proficiency exam with a mean of 375 and a standard deviation of 75. A student must score at the 40th percentile or higher to be considered "not left behind". What score should schools be aiming for in order for their students to be in good standing?

The 40th percentile is Z = -.25 so

-.25= (score – 375) / 75 solving gives you a score of 356.25 so about a 357