FINISH Chapter 14

The Basic Probability Rules

1. A probability is a number between 0 and 1 or the probability of some event is $0 \leq P(A) \leq 1$, probabilities are never negative and never greater than 1.

2. The "something has to happen rule" or more formally, a sample space S is the set of all possible outcomes. The sum of all possible outcomes must equal 1 or P(S) = 1

3. The "complement rule" is the probability that an event does not occur is 1 minus the probability that it does occur. $P(A^c) = 1 - P(A)$ or the probability that something does occur P(A) is $1- P(A^c)$ the probability that it does not occur

4. The addition rule: If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities. P(A or B) = P(A) + P(B) this also means that the two events A and B are MUTUALLY EXCLUSIVE ("disjoint")

5. The multiplication rule: If two events have common outcomes but do not influence each other, they are independent. For example, I roll two die, one die should not have an effect on the roll of the second die. In these situations you are trying to figure out the chances of two things happening together. The chance that two things will happen equals the chance that the first will happen **multiplied** by the chance that the second will happen. P(A and B) = P(A) * P(B)

EXAMPLE:

Suppose you get a job as a "assisted sales-person" in a large electronics store (like Best Buy) and prospective buyers behave this way 60% of the time they don't want anything to do with you and 40% of the time they will talk to you. Three people are walking towards you, assume they don't know each other and have INDEPENDENTLY chosen to walk towards you. What are the probabilities of 0, 1, 2, or all 3 talking to you?

Outcome	All 3 talk	2 talk to you	1 talks to you	0 talk to you
Probability	$.4^3$	(.4*.4*.6)+ (.4*.6*.4)+ (.6*.4*.4)	(.6*.4*.6)+ (.6*.6*.4)+ (.4*.6*.6)	$.6^3$

Suppose that if you can get at least two people talking to you, you are guaranteed a sale. What is your chance of a sale given these probabilities?

1. Chapter 15: Contingency Tables Again

Recall this example from the second lecture

Q. Do you think things in this country are generally going in the right direction or are they seriously off on the wrong track?

	Men	Women	Total
Right Direction	331	214	545
Off on the wrong track	285	421	706
Don't Know	46	55	101
Total	662	690	1352

Same Numbers converted to proportions

	Men	Women	Total
Right Direction	.245	.158	.403
Off on the wrong track	.211	.311	.522
Don't Know	.034	.041	.075
Total	.490	.510	1.0

2. The General Additional Rule

We use the general addition rule when events are NOT DISJOINT (in other words, there is overlap or INTERSECTIONS). The rule basically says that we add the probability of two events and then subtract out the probability of their intersection.

P(A or B) = P(A) + P(B) – P(A and B)

So in the example above, we could ask, what is the probability of selecting a MAN (call this event A) OR someone who thinks the country is OFF ON THE WRONG TRACK (call this event B). Because these are not disjoint (they have some overlap) the probability is

P (Man or Off on the wrong track) = .490 + .522 – .211

If we didn't subtract out the "overlap" we would be double counting.

3. Conditional Probability (pp 287-288)

Definition: a probability that takes into account a given condition (like a pre-existing condition) is a conditional probability.

What if you knew in advance that a randomly selected American was a male? Would that change your chance of selecting an American who feels the country is in the right direction? Absolutely.

P(right direction | male) = .245/.490 = .50

The vertical bar means "given" so we interpret the symbols as "the probability of right direction given that the American is a male).

More formally

P(B|A) = P(A and B) / P(A) (p. 288 of your text) -- we can check this because

.50 = (331/1352) / (662/1352) = 331/662

4. The General Multiplication Rule (pp. 288-289)

Chapter 14's rule requires independence. The general rule is a restatement of the conditional probability equation.

P(A and B) = P(A) * P(B|A) – again, you can check this -- .245 = .49 *.50

this is equally true:

P(A and B) = P(B) * P(A|B)

Check it: .245 = .403 * (.608) where .608=(.245/.403)

5. Independence (pp. 289-290)

Independence means that the outcome of one event does not influence the probability of second event occurring. Formally, Events A and B are independent if:

P(B|A) = P(B) (recall the conditional probabilities above)

when events are independent, this is the what the multiplication rule looks like:

P(A and B) = P(A) * P(B) or P(A and B) = P(B) * P(A)

if they are NOT independent then

P(A and B) = P(A) * P(B|A) or P(A and B) = P(B) * P(A|B)

Example: In the sample of 1,352 Americans, .490 are men and .403 (545/1352) of the Americans think the country is headed in the right direction.

The question is this: are gender and direction independent?

If P(male and right direction) is equal to P(male) * P(right direction) then yes, otherwise no.

So we can check, we know that P(male and right direction) = 331/1352 = .2448 or about .245

And P(male) = .490 and P(right direction) = .403

but .490 * .403 = .197

Interpretation: if gender and direction were independent, the probability of selecting an American male who thinks the country is headed in the right direction would only be .197 (about 20%). Instead it’s closer to 25% (.245) so it seems that men are somewhat more likely to feel that the country is doing alright.

If gender and direction were independent, this is how the responses might be generated. Compare this table to the previous one and please note the differences.

	Men	Women	Total
Right Direction	267	278	545
Off on the wrong track	346	360	706
Don’t Know	49	52	101
Total	662	690	1352