

Form I

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1. It is a family tradition: your professor goes to Las Vegas every year for Thanksgiving. A new casino has opened and they are offering a modified roulette game that has 40 different numbers that are on a large wheel. The wheel is spun and one number is selected at random when the wheel stops. To play, you bet \$8 and you get to choose 12 different numbers. If the wheel lands on any one of your twelve numbers, you win \$12. If the wheel does not land any one of your 12 numbers but on any one of 8 "special numbers" you do not win or lose any money. However, if the wheel lands on any one of the remaining numbers (neither the 12 you chose nor the 8 special numbers), you lose your bet of \$8.

- a. Please calculate the expected value of this game. (5 points)

Ch. 16

OUTCOME	+12	0	-8
PROBABILITY	$\frac{12}{40} = .30$	$\frac{8}{40} = .20$	$\frac{20}{40} = .50$

$$\begin{aligned}
 \text{Expected Value} = \mu_x &= (12)(.30) + (0)(.20) + (-8)(.50) \\
 &= 3.6 + 0 - 4.0 \\
 &= \boxed{-.40}
 \end{aligned}$$

- b. Please calculate standard deviation of this game. (10 points)

Assume A is correct $\sigma_x = \sqrt{\sum_{i=1}^n (x_i - \mu)^2 (p_i)}$

$$\sigma_x = \sqrt{(.3)(12 - -.4)^2 + (.2)(0 - -.4)^2 + (.5)(-8 - -.4)^2}$$

\uparrow probability \uparrow outcome \uparrow expected value

$$= \sqrt{75.04}$$

$$= \boxed{8.6625}$$

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c. After hearing all of this talk about gambling, you decide to spend \$72 and place 9 bets (Assume that you can treat your 9 bets as if they were a random sample of size 9 and you can assume the 9 bets are a large enough sample and the bets are independent). What is your chance of walking away with your original \$72 or even more money after making 9 bets at this game? (10 points)

If after 9 bets you walk away w/ \$72 - you didn't lose any money therefore if $n=9$ like a sample



$$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{0 - (-.4)}{8.6625/\sqrt{9}} = .14 \text{ so the area is}$$

$$1 - .5557 = \boxed{.4443}$$

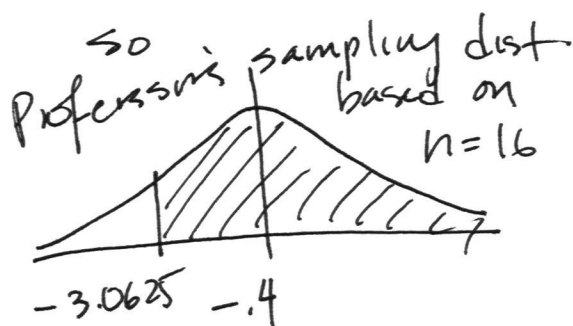
d. Suppose your professor decides to spend \$128 by placing a total of 16 bets and she loses a total of \$49. Assume that you can treat her 16 bets as if they were a random sample of size 16, that the bets are independent, and you can assume the 16 bets are a large enough sample.

Your professor's mother is a serious gambler and she decides to play too but she spends \$288 by placing a total of 36 bets (you can treat her 36 bets as a random sample of size 36 and it is also large enough and independent). However, she lost a total of \$64.

Please calculate the chance that your professor could lose 49 or fewer dollars after playing this game 16 times and that her mother could lose 64 or fewer dollars after playing the same game 36 times. Please show all of your work and then answer this question - based on your calculations, which is the luckier lady? (Let us define "lucky" as having had the smaller chance of losing as little money or less than what they lost, not necessarily which lady had the lower dollar amount lost) Please show your work (20 points)

If the professor had a total loss of \$49 after 16 tries her average is $-49/16 = -3.0625$ (losing $\frac{49}{16}$ per bet)

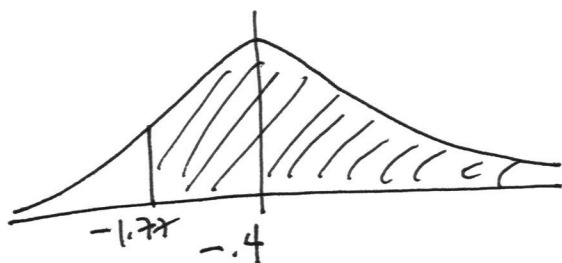
And her mom's average was $\$64/36 = -1.777$ per bet



$$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{-3.0625 - -.4}{8.6625/\sqrt{16}} = -1.23$$

$$\text{chance is } 1 - .1093 = .8907$$

Her mom's based on $n=36$



$$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{-1.777 - -.4}{8.6625/\sqrt{36}} = -.95$$

$$\text{chance is } 1 - .1711 = .8289$$

mom has the smaller chance

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2. A recent psychological study selected 841 Americans ages 17-64 at random. Of these, 169 were students enrolled in universities and colleges across the U.S. Among the students, the study revealed that 5 percent felt suicidal; 62 percent felt hopeless; and 52 percent felt depressed. Fourteen of the 169 were currently in therapy. However, 11 percent reported feeling happy. The average age of college students in this study was 22.6 years with a standard deviation of 7.1 years.

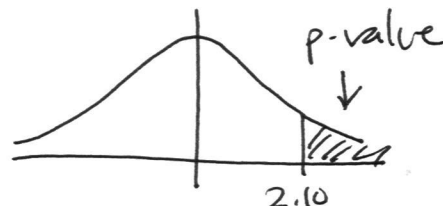
Universities and colleges across the U.S. have previously reported an average 18 percent increase (the standard deviation was 6%) in the number of students receiving psychological services. Furthermore, 60 percent of students felt hopeless and 44 percent felt depressed. The average age of college students nationally is 23.4 years with a standard deviation of 5.6 years. Age is not normally distributed.

- (a) Please test the hypothesis that the percentage of college students who feel depressed is increasing. A complete answer to this problem requires a null hypothesis, an alternative hypothesis, the correct test, the resulting p-value from the test, and an interpretation of the p-value in simple language (i.e. do you reject or not reject the null and is the result statistically significant and a brief explanation of the meaning in plain language). Please use an $\alpha = .05$ as your decision rule. (25 points)

$$H_0: p = .44$$

$$H_1: p > .44$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.52 - .44}{\sqrt{\frac{(.44)(.56)}{169}}} \approx 2.10 = Z$$



so p-value is .0179 and .0179 < .05

so REJECT THE NULL, THIS RESULT IS STATISTICALLY SIGNIFICANT WHICH MEANS There is enough evidence here to support the claim that the % depressed is increasing

- (b) Given the information above, is it possible to construct a 99% confidence interval for the population percentage (or population proportion) of college students who reported feeling happy? (10 points)

YES

NO (circle one, one point)

$np > 10$ $nq > 10$
random sample

If your answer is "yes", please construct the confidence interval in the space below. If your answer is "no", please briefly explain why it is not possible. If you circle "no" but decide to construct a confidence interval anyway in an attempt to cover your bases, you will receive a zero for your efforts (nine points)

$$\hat{p} \pm Z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .11 \pm 2.57 \sqrt{\frac{(.11)(.89)}{169}}$$

2.58
accepted

$$\approx .11 \pm .062$$

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- (c) Given the information above, is it possible to construct a 99% confidence interval for the population percentage (or population proportion) of college students who have felt suicidal?

YES

NO

(circle one, one point)

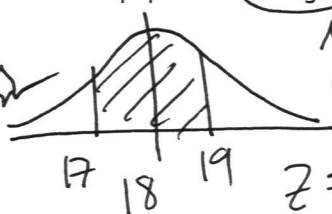
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① $np = 169 \times .05 = 8.45$ so $np < 10$

② therefore the Central Limit theorem is not applicable and the sampling distribution won't be NORMAL (can't use z scores)

- (d) What is the chance that for a new sample of size 169, the average percentage increase in the number of students receiving psychological services will be within ± 1 percent of the reported population average of 18 percent? (10 points)

Ch. 18
sampling
distribution



$$\mu = 18 \pm 1$$

$$\sigma = 6$$

$$z_{19} = \frac{19 - 18}{6/\sqrt{169}} = +2.17$$

$$z = \frac{x - \mu}{\sigma/\sqrt{n}}$$

$$z_{17} = \frac{17 - 18}{6/\sqrt{169}} = -2.17$$

the area between ± 2.17 z scores is $\approx 97\%$

- (e) The (unethical) researcher in charge of a recent study of 121 college students would like every proposed null hypothesis rejected at the $\alpha = .05$ level for any one-sided test performed. What minimum sample size should he select to be assured that he will always "reject the null" whenever his sample proportions are within 3 percent of his null hypotheses? You may assume that either $p = .50$ or $p\text{-hat}(\hat{p}) = .50$ to help you solve this problem. (10 points)

for a one sided $\alpha = .05$ this implies



so $z = 1.64$ or 1.65

sample size $n = \frac{z^2 (p)(q)}{(MoE)^2} = \frac{(1.64)^2 (.5)(.5)}{(.03)^2}$

≈ 748

or if used $z = 1.65$

$n \approx 757$