Ying Nian Wu : ywu@stat.ucla.edu

Office Hours : Tue / Thur 3:50 - 4:50 pm, Math sci 8971

Coursework : 50% weekly homework + 50% Final + Attendance

Learning material : Lecture notes + Videos on Canvas

Why probability ?

Maxwell : "The true logic of the world is in the calculus of probalities"

Quantum physics , Statistical physics ,

Artificial intelligence , machine learning / stats / data science,

monte carlo computing

Topics :

Basic concepts , axioms ,

1 random variable , discrete & continuous

2 random variables , correlation & regression , conditioning

3 + random variables , conditional independence

limiting theorem (∞ many random variables independent)

stochastic processes (∞ many random variables dependent)

$\boxed{\text{Part 0 : Basic Concepts}}$

equally likely sampling

Basic setup : Experiment $\longrightarrow$ outcome $\longrightarrow$ number

Canonical example 1 : Randomly sample a person from a population

Sample space : Ω = { all people in population } = population

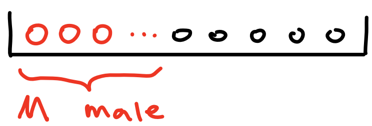
Events : A , B , C , ... $\subset \Omega$ subset of population

ex. A : the person is male (logic statement)

= male sub-population (subset)

$P(A)$ = probability of A event

ex. Ω = N balls



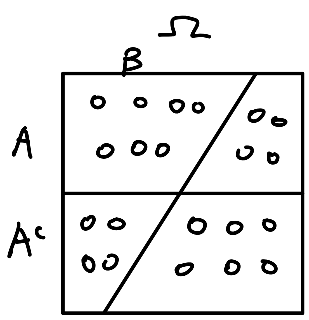
$P(A) = \frac{M}{N} = \frac{|A|}{|\Omega|}$

Axiom 0 : $P(A) = \frac{|A|}{|\Omega|}$ under equally likely sampling

B : height of person taller than 6 ft (statement)

= { people taller than 6 ft. } (subset)

$P(B) = \frac{|B|}{|\Omega|}$



Relations :

Not A , A^c , A complement

A and B, $A \cap B$ ⇒ tall males

A or B , $A \cup B$ ⇒ either male or tall

Random Variables

For a person $w \in \Omega$

$X(w)$ = gender = $\begin{cases} \text{male} & 1 \\ \text{female} & 0 \end{cases}$ (discrete)

$Y(w)$ = height (continuous)

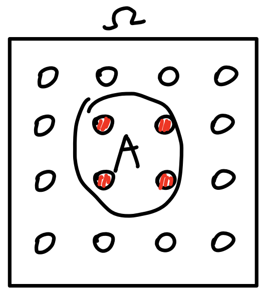
(capital : X , Y , Z , ... random variables)

$A = \{w : X(w) = 1\}$

$P(A) = P(\{w : X(w) = 1\})$ function

$= P(X = 1)$ variable

Sampling a population

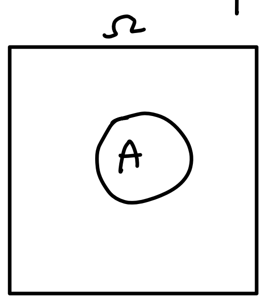


$P(A) = \frac{|A|}{|\Omega|}$

Under random sampling,

prob = population proportion

Canonical example 2: randomly sample a point from a region



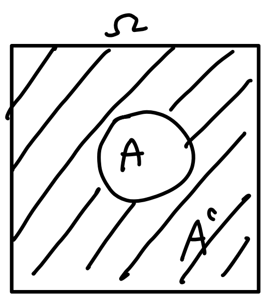
all points in Ω are equally likely uniform sampling

$\Omega = \{$ all points in $\Omega \}$
↓
uncountably infinite population

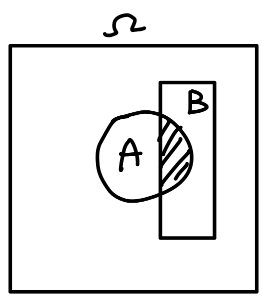
Event A : point falls into $A \subset \Omega$

Axiom 0 : $P(A) = \frac{|A|}{|\Omega|}$ – area of A / – area of Ω = proportion of A in Ω

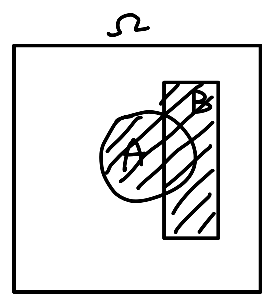
Relations



Not $A = A^c$



A and $B = A \cap B$



A or $B = A \cup B$

Axiom 0: $P(A) = \frac{|A|}{|\Omega|}$ – measure : count , length , area , volume

Axiom 1 : $P(\Omega) = 1$

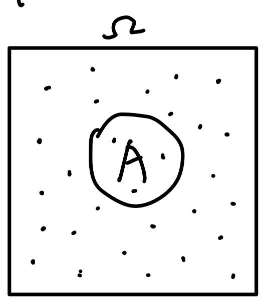
2 : $P(A) \geq 0 \quad A \subset \Omega$

3: $A \cap B = \phi$ $\quad P(A \cup B) = P(A) + P(B)$ additivity

mutually exclusive / disjoint

Long Run Frequency:

repeat n times sampling from n



As n goes to ∞:

$$\frac{m}{n} \xrightarrow[\infty]{n} \frac{|A|}{|\Omega|} = P(A)$$

subjective uncertainty of sampling once

objective frequency of n reporting

law of large number theorem