Generative Models

- Recall classifier based on CNN
- Discriminator - tell if image face or not
  - Bottom-up structure
    \[ h^l_{ij} = \sum_{oi,oj} w^l_{oi,oj} h^{l-1} + b^l + \text{rectify} \]
- Thought vectors at each layer
  - More complex
  - See bigger patterns

- Top down
- Generator
  - Can generate images
  - Still a CNN

- Generative adversarial networks (GAN)

- Discriminator
  \[ D(x) = \Pr(y=1 \mid x) \]
- Generator
  \[ \tilde{x} = G(z) \quad z \sim \mathcal{N}(0, I) \]
Maximum likelihood for D:
\[
\frac{1}{n} \sum_{i=1}^{n} \log \pi(y_i=1|x_i) + \frac{1}{n} \sum_{i=1}^{n} \log \left(1 - \pi(y_i=0|x_i)\right)
\]

= \frac{1}{n} \sum_{i=1}^{n} \log \pi(x_i) + \frac{1}{n} \sum_{i=1}^{n} \log \left(1 - \pi(x_i)\right)

\text{want to assign high probs to real examples}
\text{want to assign low probs to fake examples}

\[V = \frac{1}{n} \sum_{i=1}^{n} \log \pi(x_i) + \frac{1}{n} \sum_{i=1}^{n} \log \left(1 - \pi(x_i)\right)\]

Learn D by maximizing value (log likelihood)
Learn G by minimizing value (log likelihood)

Adversarial Game (Zero sum)
\[
\min_{G} \max_{D} \ V \\
\text{reach Nash equilibrium so sol'n to min max = sol'n to max min} \\
\text{4 saddle point}
\]

\[\text{V increase } \pi(G(x_i)) \text{ to fool } D \rightarrow \text{ tell apart fake & real examples} \]

\[\text{increase prob of fake examples as real} \]

\[
\min_{G} \max_{D} \left[\frac{1}{n} \sum_{i=1}^{n} f(x_i) - \frac{1}{n} \sum_{i=1}^{n} f(G(G(z_i)))\right]
\]

\[\text{G try to max this} \]

\[\text{D try to min this} \]

\[\text{generate fake examples} \rightarrow \text{criticize fake examples, high score to real examples, low score to fake examples} \]

\[\text{to get high score} \]

\[\text{regularized} \]
Generator

$X = g(Z)$

$Z$: latent code / embedding

encoder $z = \Phi(x)$

decoder $X = \theta(z)$

auto-encoder $z_i$

loss function

$\min_{\theta, \phi, \mathbf{h}} \frac{1}{n} \sum_{i=1}^{n} \| x_i - \theta(\Phi(x_i)) \|^2$

$z$ is much lower dimension than $x$

bottleneck

dimension reduction

manifold assumption

$X = g(Z)$

prob model

$z \sim N(0, I)$

$x = g(z) + \epsilon, \epsilon \sim N(0, \sigma^2 I)$

$x$ (weights)

$p(x|z) \sim N(g(z), \sigma^2 I)$

projection

$p(z|x)$

marginal distribution of $x$

$z \sim N(\Phi(x), \Sigma)$

$\Phi(z|x) \sim P(z|x) \sim N(0, I)$

$\epsilon \sim N(0, \sigma^2 I)$

encoder

$x$

decoder $X = g_0(Z) + \epsilon, \epsilon \sim N(0, \sigma^2 I)$
Variational Auto-encoder (VAE)

\[
\max_{\Theta, \phi} \frac{1}{n} \sum_{i=1}^{n} \left[ \log p_{\theta}(x_i) - D_{KL}(q_{\phi}(z_i|x_i) \mid \mid p_{z}(z_i|x_i)) \right]
\]

\(D_{KL}(q(z|x) \mid \mid p(z|x))\): Kullback-Leibler divergence

Evidence lower bound (ELBO)