Stats 200A

9/22

Prof Ying Nian Wu ywu@stat.ucla.edu

OH Tue/Thur 3:50 - 4:50 pm

Math Sci: 8971

$$\boxed{\begin{array}{ll} \text{Weekly HW} & 70\% \\ \text{Final} & 30\% \end{array}}$$

lecture notes & videos in Canvas Modules

Review w/ 100A on teaching page

Basic Concepts

1 random variable
- discrete
- continuous

2 random variable
- conditioning
- correlation/regression

3 & more
- multivariate
- conditional independence

∞ many iid
- law of large numbers
- central limit theorem

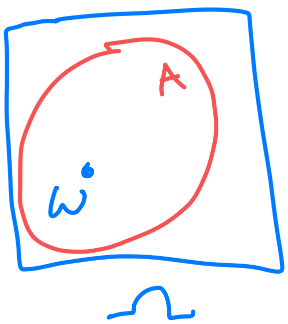
∞ many dependent
- stochastic processes
- markov
- diffusion
- SDE

Basic Concepts

sample space : Ω

attribute of outcome

$\Omega \xrightarrow[\text{random sampling}]{}$ outcome $\omega \longrightarrow$ random variable $X(\omega)$

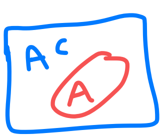
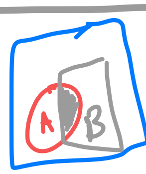
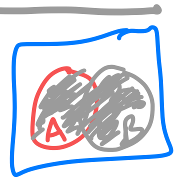


subset

event $A \subset \Omega$

$P(A)$ or $Pr(A)$: prob of A occurring

relations & notation

logic	NOT	AND	OR
set	A^c	$A \cap B$	$A \cup B$
venn			

Ex. 1 Ω is a population

ω is a person

A : male sub-population

Equally likely uniform sampling

Axiom 0 $\quad P(A) = \dfrac{|A|}{|\Omega|}$ — $|A|$: size of A; $|\Omega|$: size of Ω

counting measure

population proportion

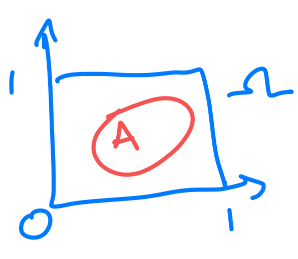
ex.) $X(w) = \begin{cases} 1 & w \text{ is male} \\ 0 & w \text{ is female} \end{cases}$

$A = \{ w : X(w) = 1$

$P(A) = P(\{ w : X(w) = 1 \}) = P(X = 1)$

$Y(w) = \text{height of } \omega$

Ex. 2 Ω is a region (unit square)



randomly sample (uniform) a point w from Ω

$$P(A) = \frac{|A| \text{ - area}}{|\Omega| \text{ - area}}$$ proportion of A w/in Ω

Axiom 1 $P(\Omega) = 1$ → generalization of Axiom 0

Axiom 2 $P(A) \geq 0$

Axiom 3 if $A \cap B = \emptyset$ (empty),
$P(A \cup B) = P(A) + P(B)$ additivity

infinite additivity

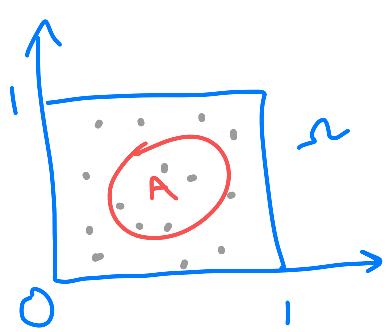
$A_1 \quad A_2 \quad \ldots \quad A_i \quad \ldots$

$A_i \cap A_j = \emptyset$ if $i \neq j$

$$P\left(\bigcup_{i=j}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Frequency

repetitions



1 million repetitions

$n(A)$ = # of points in A

$$\frac{n(A)}{n} \quad n \to \infty \Rightarrow P(A) = \frac{|A|}{|\Omega|}$$

limit definition of probability

with this limit definition, it covers the case when no points enter A or all in A

convergence in probability

$$P\left(\left| \frac{n(A)}{n} - P(A) \right| < \varepsilon \right) \to 1$$

$$\underbrace{P\left(\left| \frac{n(A)}{n} - \frac{|A|}{|\Omega|} \right| < \varepsilon \right)}_{B \subset \Omega^n} \to 1$$

$\to$ weak law of large numbers

$$P(B) = |B|$$

imagine as $2n$ dimensional cube. This cube still has volume $= 1$.

$$w_1 \quad w_2 \; \ldots \; w_n \in \Omega$$

$$w^n = (w_1, w_2, \ldots w_n) \subset \Omega^n$$

2D point

as $n \to \infty$ vol of B takes up whole vol of Ω

$$n(A, w^n) = \sum_{i=1}^{n} 1(w_i \in A)$$

$$B = \left\{ w^n : \left| \frac{n(A, w^n)}{n} - P(A) \right| < \varepsilon \right\}$$

Strong law of large numbers

$$P\left(\underbrace{\boxed{\frac{n(A)}{n} \to P(A)}}_{B}\right) = 1$$

almost sure convergence

$$B \subset \Omega^\infty = \{w^\infty = (w_1 \ w_2 \dots)\}$$

infinite D cube

$$B \subset \Omega^n$$

$$|B \times \Omega \times \Omega \times \dots \times \Omega \times \dots| = |B|$$

Equally likely

ex.) flip a fair coin

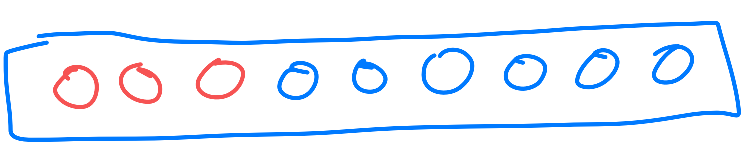
$$\Omega = \{H, T\}$$

$$P(H) = P(T) = \frac{1}{2}$$

ex.) biased coin

$P(H) = .3$ $P(T) = .7$

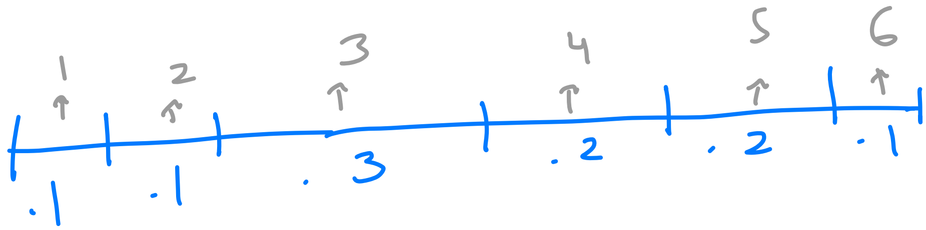
$P(\text{red}) = .3 = P(H)$



Random Variables

Discrete

x	1	2	3	4	5	6
$P(x)$	.1	.1	.3	.2	.2	.1

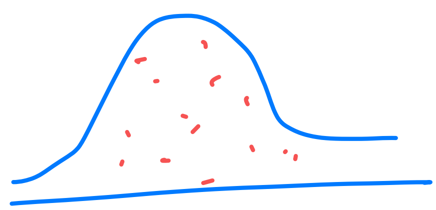


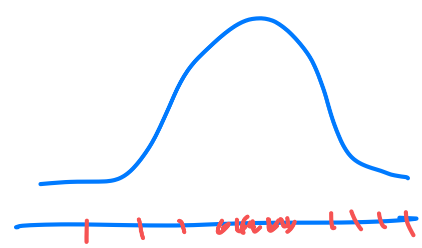
$\omega \sim \text{Unif}(\Omega)$

$X(\omega)$

Continuous

$X \sim f(x)$





$P(X \in A) = |A|$