Problem 1
a) \[\frac{d}{dt} \int_0^t y \, dt \]
\[y = \frac{1}{2} \cdot \text{center} \cdot y = \frac{d}{2} \quad y = 0\]
compute area of A by integral
\[y \leq \frac{d}{2} \sin \theta\]

b) \[\frac{1}{\pi} \quad E \left( \frac{1}{\pi} \right) \quad \text{Var} \left( \frac{1}{\pi} \right) \quad \text{sd} = \sqrt{\text{Var} \left( \frac{1}{\pi} \right)}\]

Problem 2
\[P \left( T \in (t, t+\Delta t) \right) = \frac{t}{t} \int_0^t \left( 1 - \lambda(t) \Delta t \right) \lambda(t) \Delta t\]
approximation\[= \int_0^{t+\Delta t} e^{-\int_0^t \lambda(t) \, dt} \lambda(t) \Delta t = e^{-\int_0^t \lambda(t) \, dt} \lambda(t) \Delta t\]
\[-e^{-\int_0^t \lambda(t) \, dt} \lambda(t) \Delta t \]
doesn’t happen until after t
Survival analysis

\[ \lambda(t) \quad \lambda_0(t) \]

Baseline rate

\[ \log \left( \frac{\lambda(t)}{\lambda_0(t)} \right) = X^T \beta \]

(age, gender, treatment)

Problem 3

\[ n \text{ layers} \]

\[ X = Y(1) + (n - Y)(-1) \]

\[ = (2Y - n) \Delta x \]

\[ F(x) = 0 \]

\[ \text{var}(X) = 4 \Delta x^2 \quad \text{var}(Y) = n \Delta x^2 = \frac{t}{\Delta t} \Delta x^2 \]

\[ = + \left( \frac{\Delta x^2}{\Delta t} \right) = \sigma^2_t \quad \sigma^2 = \frac{\Delta x^2}{\Delta t} \quad \Delta x = \sigma \sqrt{\Delta t} \]
velocity = \frac{\Delta x}{\Delta t} = \frac{\delta \sqrt{\Delta t}}{\Delta t} = \frac{\delta}{\Delta t} \quad \Delta t \to \infty

\begin{align*}
X(t + \Delta t) &= X(t) + \xi(t) \Delta x \\
\xi(t) &= \begin{cases} 
1 & \text{with probability } \frac{1}{2} \\
-1 & \text{with probability } \frac{1}{2}
\end{cases}
\end{align*}

\begin{align*}
P_{t+\Delta t}(x) &= \left( P_t(x - \Delta x) + P_t(x + \Delta x) \right) / 2 \\
\frac{P_{t+\Delta t}(x) - P_t(x)}{\Delta t} &= \frac{\left( P_t(x + \Delta x) - P_t(x) \right) - \left( P_t(x) - P_t(x - \Delta x) \right)}{2\Delta t} \\
&= \frac{2\left( P_t(x + \Delta x) - P_t(x) \right)}{2\Delta x} - \frac{P_t(x) - P_t(x - \Delta x)}{2\Delta x} \\
&= \frac{\delta^2}{\Delta x} \frac{1}{2} \frac{\partial^2 P_t(x)}{\partial x^2}
\end{align*}

\begin{align*}
\Delta x &= \delta \sqrt{\Delta t} \\
\Delta t &= \Delta x^2 / \sigma^2
\end{align*}

\begin{align*}
\frac{d}{dt} P_t(x) &= \frac{\sigma^2}{2} \frac{d^2}{dx^2} P_t(x)
\end{align*}

partial differential equation
heat equation
distribution of \text{mil particles}
Problem 5

\[ E_p \left[ \log \frac{p(x)}{q(x)} \right] = \sum_x p(x) \log \frac{p(x)}{q(x)} \]
\[ Y = \frac{p(x)}{q(x)} \]
\[ \sum_x p(x) \frac{q(x)}{p(x)} = 1 \]
\[ E_p \left[ -\log \frac{q(x)}{p(x)} \right] = E_p \left[ -\log Y \right] \geq -\log(\mathbb{E}(Y)) \]

ex.)

<table>
<thead>
<tr>
<th>X</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>1/2</td>
<td>1/4</td>
<td>1/8</td>
<td>1/8</td>
</tr>
</tbody>
</table>

prefix code

\[ E \left( \# \text{ of coin flips} \right) = E_p \left[ -\log_2 p(x) \right] \]

code length

\[ E \left( \text{code length} \right) = E_p \left[ -\log_2 q(x) \right] \]

excess length: \( D_{KL}(p \| q) = E_p \left[ \log_2 \frac{p(x)}{q(x)} \right] \)

redundancy
Transformation of continuous RV

\[ X \sim N(0, 1) \]

\[ f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \]

density of \( X \)

\[ Y = \mu + \sigma X \]

\[ E(Y) = \mu + \sigma E(X) = \mu \]

\[ \text{Var}(Y) = \sigma^2 \text{Var}(X) = \sigma^2 \]

linear transformation

\[ y = \mu + \sigma x \]

\[ x = \frac{y - \mu}{\sigma} \]

\[ \text{variance is tight} \]

\[ \text{variance is wider} \]

\[ y = \mu + \sigma x \]
\[ y = h(x) \]
\[ y = h^{-1}(y) = g(y) \]

\[ p(x \in (x, x+\Delta x)) = P(y \in (y, y+\Delta y)) \]

\[ \int x \Delta x = f_y(y) \Delta y \]
\[ dF_x(x) = dF_y(y) \]

\[ f_y(y) = f_x(x) \frac{\Delta x}{\Delta y} \quad \text{... find density of } Y \]

Symbolically, \[ = f_x(g(y)) | g'(y) | \]

\[ X \sim f_x(x) \Delta x = f_x(g(y)) \Delta g(y) = f_x(g(y)) g(y) \Delta y = f_y(y) \Delta y \quad \text{for } Y \]

\[ \Delta x \quad \text{squeezed } \Delta x \]

\[ \text{Sketch of } y \]

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\[ f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \]

\[ X \sim f_x(x) \, dx = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) \frac{dy}{\sigma} \]

\[
\begin{align*}
\mathbb{E}(X) &= \mu \\
\text{Var}(X) &= \sigma^2 \end{align*}
\]

\[ y = \mu + \sigma \\
x = \frac{y - \mu}{\sigma} \]

\[ e^{-\frac{(y-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \]

\[ F_x(x) = \frac{1}{2} \left(1 + \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right) \]

\[ F_y(y) = \frac{1}{2} \left(1 + \text{erf}\left(\frac{y-\mu}{\sqrt{2}\sigma}\right)\right) \]

\[ dF_x(x) = dF_y(y) \]

\[ x = g(y) \]

try to find transformation to map

\[ y = h(x) \]

monotone mapping

preserve order

if \( x \) is 100th value \( \rightarrow \) should be 100th value in \( Y \)

same area
you can transform one rv to another rv

\[ y = F_{y}^{-1}(F_{x}(x)) \]

\[ y = h(x) \]

\[ X \sim \text{Unif}[0,1] \]

\[ y = F^{-1}(u) \]

\[ F_{u}(u) = u \]

prob of \( u \leq u \)

sample \( u \in [0,1] \) vertically and then look horizontal, find \( u \) on vertical, then get \( x \)

\[ F'(x) = f(x) \]

slope \( \Rightarrow \) density

given a percentile of a random percentile (Uniform)