Chapter 3. From student to celebrity: 1949-1952

The challenge of adequately describing the years that Alexandre Grothendieck spent as a doctoral student in Nancy is considerable, for during those three years he underwent some of the most tremendous experiences that life can bring to a young man, while simultaneously making a series of extraordinary mathematical discoveries: thus, the time has come to weave a real, albeit informal, discussion of mathematics into the fabric tale of Grothendieck's life.

In terms of his previous education and reputation, Grothendieck was infinitely removed from the family of young French talents so close to him in age, Ecole Normale students spotted early on and nurtured all along their way to success: Pierre Samuel, Roger Godement, René Thom, Jacques Dixmier, Jean Cerf, Yvonne Bruhat and her brother François, Jean-Pierre Serre, Bernard Malgrange. He was so completely unknown to this group and to their professors, came from such a deprived and chaotic background, and was, compared to them, so ignorant at the start of his research career, that his fulgurating ascent to sudden stardom is all the more incredible; quite unique in the history of mathematics.

Two startled professors: Fall 1949

An anecdote survives about Grothendieck's arrival in Nancy: the story of his rude reception at the hands of Dieudonné when, on their very first contact, he showed him a dense handwritten manuscript on "generalized integrals". He had already mentioned this work in writing to Dieudonné, and had received a warm and friendly response in which Dieudonné praised his "ardor for mathematics". But Dieudonné's initial receptiveness did not outlast a first look at the actual text. Those who recall this incident (or rather, who recall Dieudonné's telling them about it) claim that Dieudonné gave Grothendieck a rather sharp dressing down, finding that the work displayed a reprehensible tendency to gratuitous generality. Whether or not his assessment is justifiable, it is clear in any case that he immediately seized the essence of Grothendieck's approach – and was quick to discourage it. As Schwartz recounts it in his autobiography:

> *Il présenta d'abord à Dieudonné un article d'une cinquantaine de pages, sur "L'intégration à valeurs dans un groupe topologique". C'était exact, mais rigoureusement sans aucun intérêt. Dieudonné, avec l'agressivité (toujours passagère) dont il était capable, lui passa un savon mémorable, arguant qu'on ne devait pas travailler de cette manière, en généralisant pour le plaisir de généraliser. Il fallait que le problème traité soit difficile, et susceptible d'applications dans le reste des mathématiques (ou d'autres sciences); ses résultats ne serviraient jamais à rien ni à personne.* [Sch 292][1]

The irrepressible Grothendieck does not seem to have conceived the slightest grudge towards Dieudonné on this account; perhaps his own family experiences with aggressivity and anger made him dismiss such scenes easily, or perhaps the scolding tone was not enough to mask the underlying warm generosity and benevolence that Dieudonné was

noted for, as much or more than for his brusque manner and frequent loud explosions of temper. Quite probably, Grothendieck simply accepted the rebuke as valid, cast aside his past work without a thought as he had already done twice before, and rolled up his sleeves to have a new stab at proving his worth, by working for the first time on problems set to him by someone else.

In order to direct the young neophyte towards more concrete questions, Dieudonné and Schwartz handed him a copy of their most recent paper*, *La dualité dans les espaces* $\mathcal{F}$ *et* $\mathcal{LF}$. This article was an attempt to transport some major results in the theory of Banach spaces (relations between different topologies and elements of the duality theory) to more general locally convex topological vector spaces, essentially for the purpose of developing Schwartz's recent and exciting theory of distributions, for which he would win the Fields Medal the following summer.

Interestingly, one of the new features of the Schwartz-Dieudonné article was the fact that they built their $(\mathcal{LF})$-spaces by taking inductive limits of $(\mathcal{F})$-spaces. At that time, the ideas of projective and inductive limits were relatively new; projective limits were used in specific situations for specific constructions, without recourse to a general definition, while inductive limits had, seemingly, only just made their first appearance, without yet being recognized as a general procedure (and even less as the dual procedure to projective limits), for the purpose of defining a topology on the space $\mathcal{D}$ of C^∞ functions with compact support studied by Schwartz. Later, Grothendieck and others would adopt and formalize the definitions of these limits, essentially in the context of homological algebra, thus making them into concepts uniformly applicable to a great many different mathematical situations rather than just tools used once at a time in each context. This is just one of the features of his early work in functional analysis which connects it in a deep way to his later body of work. Because he left the subject of topological vector spaces in 1955 completely and forever, it is often considered as something of a separate episode in his mathematical life. But in fact, many of the mathematical seeds of his later work were sown during his functional analysis period, and what is more, many of the traits that later came to characterize his inimitable style were already present there as well. We will explore some of these in more detail below.

The Dieudonné-Schwartz article ended with fourteen questions that the authors had not been able to solve, all concerning desirable basic properties of $(\mathcal{F})$-spaces, their duals, their topologies, their subsets, and the linear functionals on them. Dieudonné handed the article to Grothendieck with the injunction to choose a few and think about them. As Schwartz recounts it, Grothendieck disappeared totally for some weeks, after which he returned with profound and difficult solutions to several of them. Jacques Dixmier recalls Dieudonné bounding into a Bourbaki meeting with his usual energy and telling everyone how a new student had shown up in Nancy with some useless manuscript about integrals and had earned what Dieudonné felt was a well-deserved rebuke – but then had gone off and solved the bulk of the problems at the end of his recent paper with Schwartz. Dieudonné was frankly impressed, and only too pleased to share his enthusiasm with anyone who would listen.

Just weeks after Grothendieck's arrival in Nancy, Dieudonné was considering him as a

---

* *Ann. Inst. Fourier Grenoble* **1** (1949), 61-101.

suitable recruit for Bourbaki*. Grothendieck was surprised and delighted by Dieudonné's eagerness: "Si j'ai vu un mathématicien faire usage d'un puissant et élémentaire "pouvoir d'encouragement", c'est bien lui! Je n'y ai jamais resongé avant cet instant, mais je me souviens que c'est dans ces dispositions aussi qu'il avait accueilli déjà mes tout premiers résultats à Nancy, résolvant des questions qu'il avait posées avec Schwartz (sur les espaces $\mathcal{F}$ et $\mathcal{LF}$). C'étaient des résultats tout modestes, rien de génial ni d'extraordinaire certes, on pourrait dire qu'il n'y avait pas de quoi s'émerveiller."[2]

In spite of this modest assessment of his own work, Schwartz and Dieudonné did marvel. Grothendieck's solutions to these problems gave rise to his first publication, a brief mathematical note submitted to the *Comptes Rendus de l'Académie des Sciences* in 1950, only a few months after he first set foot in Nancy.

Although Grothendieck later gained something of a reputation for disliking to read papers by other mathematicians, his early work in Nancy shows that he was absorbing the notions and the known results in functional analysis with a speed bordering on bulimia. A second note appeared only a few months after the first one, containing a generalization of a recent result of W. Eberlein. Interestingly, the purpose of his generalization, its actual motivation, was the production of some "pathological" examples of phenomena in Dieudonné and Schwartz's $(\mathcal{LF})$-spaces; essentially counterexamples to some of the other questions asked in their paper; oddly enough, this, also, is in contrast with the reputation that he later held for being ill-adapted to search for counterexamples to beautiful-sounding statements. He continued this work on locally convex topological vector spaces with a third note, published in the fall of 1950, and a fourth, in 1951, which completed the solutions of the entire original list of fourteen problems. It seemed clear at that moment that this collection of results would form his doctoral thesis. In a letter from Chicago dated April 10, 1951, Dieudonné wrote: "Je conçois que vous soyez un peu déçu de voir que si peu de questions admettent une réponse positive, mais votre thèse n'en sera pas moins un travail très précieux en apportant tout au moins *une* réponse à des quantités de questions qu'il était naturel de se poser, et qui demandaient beaucoup d'ingéniosité pour les résoudre; d'ailleurs, vous avez tout de même un bon nombre de théorèmes qui à mon avis sont les seuls résultats profonds de la théorie depuis Banach, et vous n'avez pas à vous inquiéter sur la valeur de votre thèse; mais je suis bien d'accord avec vous pour penser qu'après cela vous aurez définitivement tué le sujet."[3]

Towards the end of 1950, Grothendieck wrote a letter to Jacques Dixmier, asking him for input on some very specific questions concerning fixed points under the action of groups of a certain type acting on a locally convex space. Not only had Dixmier, who was just four years older than Grothendieck, just defended a brilliant doctoral thesis at the Ecole Normale under Gaston Julia, but he was already the author of over a dozen published articles, and furthermore had been co-opted into the prestigious Bourbaki group, mathematicians who worked together on the grand task of constructing and writing solid

---

* Grothendieck was one of five "cobayes" (guinea-pigs; young mathematicians invited to see whether they would be suitable for Bourbaki and Bourbaki for them) at a Bourbaki meeting which took place in Nancy, Feb. 3-7, 1950. The other "cobayes" were F. Bruhat, Braconnier, Berger and Riss; the actual members were Cartan, Chabauty, Delsarte, Dieudonné, Ehresmann, Godement, Mackey, Pisot, Roger, Samuel, Serre, Schwartz.

foundations intended to form the basis for all future mathematical teaching and research. But if Grothendieck had been slightly intimidated by this type of student during his year in Paris, all such doubts had slipped away after the successes of his first year in Nancy, and apart from the formality of his tone, nothing in his letter to Dixmier indicates the slightest sense of inferiority. Rather, it is striking to note how Grothendieck had incorporated the casual Ecole Normale slang into his own writing style, he doesn't hesitate to qualify groups with a certain agreeable property as "sympathiques", and makes cheerful use of the verb "canuler", a term of local Normalian dialect. At this time, Grothendieck's style of mathematical writing partly reflects the voices of Dieudonné and Schwartz, and does not yet possess the very original ring it later developed.

Dixmier responded quickly but unsatisfactorily; his suggestions were already known to Grothendieck, and they didn't yield what he needed. On the other hand, Dixmier asked some questions about certain statements that Grothendieck had made rather lightheartedly, and received a slightly meek reply: "J'avais cru voir que ces $\phi_{x,y}$ sont en effet flot presque périodiques, mais n'en aperçois plus la raison maintenant que je vous écris, de sorte qu'il me semble bien possible que je me sois trompé – mais je n'en suis pas convaincu."[4] A little disturbed by this, or perhaps by the effect he imagines it might have on Dixmier, he adds a reassuring postscript to the end of his response: "Les autres résultats de ma précédente lettre, et de celle-ci, ont été regardé par moi avec assez de soin pour être tout à fait certains!"[5]

New friendships: 1950-1952

Although Grothendieck worked at mathematics for the greater part of every day, he was no hermit; his stay in Nancy brought him human relations of a kind he had not known before during his tormented childhood, his solitary life in a remote village with his mother as sole company, or the difficult and lonely first year in Paris. In Nancy, Grothendieck met mathematicians who, although different from him in social background, shared his passion for mathematics, treated him as an equal, and more personally, recognized the interesting and original character behind the blunt and unconventional manners.

Laurent Schwartz and his wife, mathematician Marie-Hélène (née Lévy) were the first to extend the hand of friendship to the young man who described himself as "ich bin ja so schlecht erzogen!"* He was peculiar, they agreed, and certainly the formal behavior of members of the wealthy provincial French bourgeoisie were very foreign to him, but at the same time, they found him an attractive personality, warm-hearted and fascinating. Their daughter Claudine was a little girl in the early 1950s, but she remembers her parents welcoming Grothendieck into their household like a son, and talking about him often. Finding a proper time to invite him for a meal became something of a logistical nightmare, though, as Grothendieck sometimes slept in the day and worked through the night. After pressing him with great interest to understand why he did this, Schwartz came to realize that Grothendieck seemed to function on a biological schedule of 26 or 27 hours, which led him periodically to live on the opposite schedule from everyone else.

The Schwartz family had a servant named Alice, an elderly woman who organized

* "I'm so badly brought up!", letter to Dagmar Heydorn

their household and to whom they were very attached. Alice ate together with the family, and Claudine recalls noticing even then how fond of Alice Grothendieck was, and that he made an effort which was at the same time conscious and yet very natural to extend his friendship to people of all social classes equally. Also, as Marie-Hélène soon noticed, Grothendieck felt comfortable with women who were close to his mother in age. Hanka had come to join Grothendieck in Nancy a few months after he arrived there, and was still, as she had always been, the center and focal point of the circle of his human relationships.

Grothendieck was also regularly invited to the home of Roger Godement, another brilliant young member of Bourbaki, seven years older than Grothendieck and already well-known. The Bourbaki meetings were intensive, with members basically spending 24 a days in close proximity, and a year after his first time, Grothendieck was again invited to be a cobaye, this time in the company of Glaeser and "a Brazilian". To give the flavor of that meeting, "La Tribu" (the irregular newsletter put out by members of Bourbaki to keep a record of their doings) recalls that "The Congress was run partially under the sign of restrictions. As Rockefeller's manna ceased to rain down, lunches took place in more economical venues, to our discomfort: at first the faithful, ill-nourished, paid but little attention to the debates, which frequently degenerated into personal conversations; there was then a general evening rush to the Pantagruelian meals provided by the Nancyans, causing some to stay up all night. The diet of the High Commissars made devastating progress, and the Master ordered all the faithful to cease this deviation at once, and to return to the general direction of the early years. On the advice of a philosopher, a chair in Higher Gymnastics will be created for Serre in Nancy."

Roger Godement was and remained all his life deeply involved in politics, and he recalls feeling a little impatient with Grothendieck's anarchist leanings, which struck him as undigested, inherited directly from Grothendieck's parents without much personal thought. He liked Grothendieck very much, and tried to give him the rudiments of a political education, but the overall effect that Grothendieck had on him was expressed by a succinct exclamation: "C'était un vrai sauvage!"*

It wasn't exactly his manners, which Godement described as being very agreeable; he smiled easily and seemed to have been raised with love, his voice was gentle and Godement never heard him say an unkind word. If Magnier had noticed the traces of "suffering and privation" in Grothendieck's instability, Godement saw nothing which suggested the traumas and tragedies of his childhood. It was obvious that he had very little money, and he was aware that Grothendieck's father was deceased, but he didn't imagine anything worse, and later regretted never having even thought to ask.

Grothendieck's financial situation improved at the beginning of his second academic year in the fall of 1950, when the department at Nancy nominated him for a temporary position as "Attaché de Recherche" at the CNRS. At that time, the CNRS** position of Attaché de Recherche was in fact a thesis scholarship which did not wish to call itself as such. The positions were intended to be temporary, and after completing the thesis, students were expected to move on to permanent university or teaching positions. These

---

* "He was a real savage!"

** Now "le CNRS, Centre National de Recherche Scientifique", but at the time "la CNRS", Caisse National de Recherche Scientifique".

positions were created by Frédéric Joliot, head of the CNRS in 1945, who explained his choice of terminology by saying: "The term 'scholarship recipient' does not please us. The candidates should not have the impression that they are asking for a favor, but that their payment is the salary for a activity which is primordial from the point of view of the nation. Jean-Pierre Serre recalls the ending of his time as an "Attaché de Recherche with the remark: "I would have willingly stayed on, but the morals of the time didn't leave me any choice: a mathematician who had defended his thesis had to take up a University position, and leave his [CNRS] position to younger people."

Grothendieck successfully obtained the CNRS position in the fall of 1950, and held it until his definitive departure from Nancy. This provided him and his mother with a steady, although limited income; in any case they always lived very simply. But Grothendieck was irremediably bizarre, without the slightest idea that his natural impulses might sometimes be considered rude, or at least unusual. Invited to Godement's home for lunch, he might exclaim with dismay at the plenitude of the food. "Why do you eat like that? You don't need all this food, all that meat!" Or, he would suddenly announce that henceforth, he was going to eat only milk, cheese and bread, and he would demand to be served these aliments and nothing else, after which he would sit pleasantly chatting with the family, ignoring the carefully prepared meal but perfectly content. His health must have been excellent, because the milk at that time was not pasteurized, let alone sterilized, and the death of babies due to bacteria in the milk was by no means rare. Although pasteurization was invented in France (by Louis Pasteur, obviously), it was widespread in America before becoming so in France, and Americans visiting France could get serious stomach aches from drinking the milk there. But Grothendieck gulped it down, apparently unscathed. Godement attributed Grothendieck's eating habits, and also his scrupulous cleanliness, to a continual striving after an elusive purity.

In April 1950, when Grothendieck had been in Nancy for several months already, a young student arrived from Brazil on a scholarship to spent two years studying with Dieudonné. Back in Rio de Janeiro, Paulo Ribenboim* had found a copy of notes of a course given by Dieudonné, and read them with interest. Encouraged by one of his professors, who said – almost by chance – "Go to France", Ribenboim applied for a scholarship, chose Nancy as a destination, and arrived there, inexperienced and fresh out of school, excited and uncertain of what to expect.

Certain things about France startled him. The effects of the war were still visible everywhere; he was not used to seeing people living in luxurious and beautiful buildings dressed in patched clothing and wooden-soled shoes**. In some ways, though, Paulo fit better into the circle of Nancy society that he encountered through Schwartz and Dieudonné than Grothendieck; indeed, he met there and soon married the woman who has been his wife now for over 60 years. From an upper class background, he was a well-educated young man; his French was already quite good, and he was a fine pianist. The cultured

---

* Very possibly the unnamed "Brazilian" at the Bourbaki meeting of February 1951.

** Chaim Hönig, another young student from Brazil, recalls that in visiting France in 1953, he felt embarrassed to find himself better dressed even than the professors there. On his next visit to France eight years later, the situation had changed: although he wore the same things as before, he was now the least well-dressed person around

atmosphere that prevailed in the households of university professors was comfortable and familiar to him, and like Grothendieck, he was warmly received and made welcome in the Dieudonné and Schwartz households from the start. Ribenboim recalls Dieudonné asking him what he knew on the first day that he arrived in Nancy. "Lattices..." he began, only to be told to forget about all of that at once, procure copies of Bourbaki's books, and start studying them to the exclusion of everything else. After that, Ribenboim went to Dieudonné's house every Friday to ask him the weekly harvest of questions about his reading. He came to know Dieudonné quite well during those visits, and eventually got used to his bark - although he still remembers feeling deeply embarrassed when Dieudonné flew into a rage over his teenage daughter's Latin homework that was sharper than anything that Paulo's own ignorance had ever induced!

The Schwartz family was more easygoing, and Ribenboim visited often. "You, Paulo - you're not crazy," Schwartz told him unexpectedly one day, soon after his arrival. "I have a student who's very good, but he works too much. He's coming here later. Maybe you can be friends." And indeed, Grothendieck did soon arrive, vigorously pedaling his bicycle. They talked; Ribenboim enjoyed cycling as well, and Grothendieck was curious to hear about the beauties of Brazil. They took to strolling about Nancy together, talking endlessly about everything and nothing; rarely about mathematics. Grothendieck did visit Paulo's room once, and stared thunderstruck at Paulo's enormous pile of math books. "What on earth is all that?" "That's what I'm going to read," said Paulo optimistically. But Grothendieck, not much of a believer in methodical study, merely responded "I bet you'll never read that in your whole life."

Ribenboim also discovered Grothendieck's fondness for piano music, proposed that they go to concerts together, and willingly played for him. Listening to some Debussy, Grothendieck was suddenly seized by the strong desire to play the piano that had already possessed him as a young boy in Le Chambon, and again during his year in Paris. "So go rent a piano," Paulo suggested practically. "I can't do that." "Why not?" "Look at my clothes." The sight of Grothendieck dressed summer and winter in simple and somewhat frayed shirt, pants and sandals was so familiar to his friends that they hardly noticed it any more. But Ribenboim had to agree that Grothendieck's appearance was hardly likely to inspire financial confidence in a piano salesman, and he didn't own anything else. "You go for me," said Grothendieck hopefully, so it was Ribenboim who took care of the rental contract and the delivery. A piano was duly brought to the room that Grothendieck rented in a rather stately house at 33, rue du Maréchal Gérard, and he took to practicing on it day and night. According to Ribenboim, his habits of late practice drove one landlady after another quite mad, and he ended up moving no less than 23 times before he finally left Nancy.

One of these moves brought him to a part of town rather far from the center of Nancy, in a popular district, where he rediscovered some of the habits of his youth. Once he told a boy in the street to quiet down, and the boy retorted by calling him "Sale boche!" Clearly the insult was one which Grothendieck had heard before (he kept a slight trace of his German accent all his life). He slapped the boy roundly about the head and found himself carted off to the police station by an angry officer who had witnessed the scene. He wasn't in the least bit embarrassed, however: he was physically very strong and fancied himself

to be something of a boxer. Nothing could frighten him, and for that matter, there wasn't much that could impress him, either.

A doctoral thesis: 1951-52

The year 1951 brought the solutions to the remaining problems at the end of the Dieudonné-Schwartz paper. The results were published in Grothendieck's fourth note submitted to the CRAS, and Dieudonné and Schwartz decided that Grothendieck was now in need of a really deep topic to write his thesis on.

After some discussion, it was decided that Laurent Schwartz would be the official thesis adviser. Schwartz suggested a problem that had been puzzling him for some time: the true nature of the topology that he had put on the tensor product $\mathcal{D}' \otimes F$ for any locally convex topological vector space F (where $\mathcal{D}$ denotes the space of C^∞ functions with compact support, and $\mathcal{D}'$ denotes its dual), by considering it as a subspace of the space $\mathcal{L}(\mathcal{D}, F)$ of continuous linear functions from $\mathcal{D}$ to F. While Dieudonné had realized that the topology that Schwartz had defined on $\mathcal{D}$, constructed by laboriously defining the open sets, could be interpreted more theoretically as an inductive limit, thus yielding information on the induced topology on $\mathcal{D}' \otimes F$, Schwartz still didn't understand how one could put a really natural topology on any general tensor product of locally convex topological vector spaces $E \otimes F$ which would generalize the one he had when $E = \mathcal{D}'$. This, then, was the problem that Schwartz chose for Grothendieck: simply to find a good topology on such a tensor product. It was vague enough to give plenty of scope for new discoveries.

Schwartz recalls that he went off to Brazil for the summer, expecting to begin serious work with Grothendieck on his return in the fall*. But...

Je reçus fin juillet, au Brésil, une lettre de lui, très déçue: il existait sur $E \otimes F$ deux topologies localement convexes, aussi naturelles l'une que l'autre, et différentes! Donc il n'y avait là rien d'intéressant à faire. Je ne savais quoi lui répondre. Pourtant il y avait bien sur $\mathcal{D}' \otimes F$ une seule topologie qui s'imposât. Mais difficultés ou défaites peuvent être sources de victoires. Deux semaines plus tard, je reçus un nouveau courrier triomphant, ces deux topologies coïncidaient dans le cas de $\mathcal{D}' \otimes F$. Il existe des espaces localement convexes E, qu'il appela "nucléaires", tels que, pour tout F, les deux topologies sur $E \otimes F$ coïncident. Tout devenait clair.[6] [Sch 293]

---

* In his autobiography, Schwartz gives the year of Grothendieck's arrival in Nancy as 1951 and claims he worked out the theory of nuclear spaces in the summer of 1952, and finished writing his thesis at the end of 1953. It is a fact that Schwartz was visiting Brazil in the summer of 1952, yet these dates cannot be quite right, as Grothendieck actually arrived in Nancy in October 1949, and by the end of 1951, he had published the answer to precisely the subject set him by Schwartz. Furthermore, he finished writing his thesis at the end of 1952, and defended it in February 1953. Thus it seems that Schwartz is off by one year; he must have given him the problem in the spring of 1951 and Grothendieck worked it out that summer (perhaps Schwartz visited Brazil both years).

This is precisely the question answered in Grothendieck's fifth note in the CRAS, dated December 1951. The abstract reproduced from *mathscinet* reads:

Let E and F be locally convex, linear topological spaces, and let $E \otimes F$ denote the tensor product, that is, the set of all finite linear sums of products. Then $E \otimes F$ can be so topologized that its completion $E \widehat{\otimes} F$ has for dual precisely all numerical-valued $\varphi(x, y)$ which are linear in $x \in E$ and in $y \in F$ and continuous in (x, y). Also $E \otimes F$ can be topologized so that its completion $\overline{E \otimes F}$ has for dual precisely the linear $\varphi(x, y)$ which are continuous in x and in y separately. The two completions need not coincide but there is a natural linear continuous mapping of $\overline{E \otimes F}$ into $E \widehat{\otimes} F$.

Oddly, the abstract does not mention *nuclear spaces*, which are at the heart of Grothendieck's discovery: this is the name he gave to spaces E having the property that the two topologies on $E \otimes F$ actually coincide for every F, meaning that the natural continuous linear mapping mentioned above is an isomorphism for every F (he showed that for a general space E, it need be neither one-to-one nor onto*). His theory went beyond the first salient result, that the space $\mathcal{D}'$ studied by Schwartz was nuclear**. Already in this first publication on the topic, he gave some equivalent properties characterizing all nuclear spaces; he was to spend the whole of 1952 developing the theory in "exhaustive" detail (his own word, from the summary paper he published in the *Annales de l'Institut Fourier* shortly after).

In his report on Grothendieck's submitted thesis (undated, but probably from the late fall of 1952), Schwartz describes the numerous results proven there in some detail, and concludes with words of resounding praise, tempered by a few suggestions of the type a benevolent father might address to a (mathematically) hotblooded youth***.

Le travail présenté ici est très long, mais ne contient aucun "délayage". Les théorèmes énoncés sont difficiles, et nécessitent beaucoup d'ingéniosité. Beaucoup d'idées originales, une technique parfaite (chaque démonstration est aussi courte que possible, et utilise exactement les méthodes adéquates), une conception très claire et très ordonnée (la première rédaction était à peu de choses près l'état final), tout cela constitue d'excellentes qualités. Certes il y a des défauts. Les énoncés des théorèmes sont trop longs, donc trop lourds; mieux eût valu les raccourcir, quitte à en mettre une partie en remarque ou dans la démonstration. D'autre part il manque le discernement qui permet de séparer, parmi des résultats nouveaux et difficiles à démontrer, ceux qui, pour l'avenir des mathématiques, seront secondaires ou essentiels; d'où une présentation massive, qui rendra très difficile la lecture aux non spécialistes. Ces défauts de "jeunesse", de nature plutôt pédagogique,

---

* Grothendieck uses a very English-sounding turn of phrase, writing: "L'application ... n'est ni sur, ni un isomorphisme vectoriel topologique dans." The use of "injective" and "surjective", and the use of "isomorphism" for a map having both properties, apparently came later.

** The name "nuclear" comes from Schwartz's 1950 "théorème des noyaux", which Grothendieck showed was an immediate consequence of his result when $F = \mathcal{D}'$.

*** In view of his later work, it cannot be said that Grothendieck ever made the slightest effort to follow Schwartz's advice.

ne doivent pas être négligés, mais n'enlèvent nullement à cette thèse son caractère et sa valeur très exceptionnels. Enfin, si l'auteur s'est jusqu'à présent confiné dans une branche des mathématiques, il faut dire que la moisson des résultats obtenus a justifié cette spécialisation, que M. Grothendieck a par ailleurs une solide culture, et des capacités qui lui permettront sûrement de trouver aussi dans d'autres domaines.[7]

The thesis was defended in Paris, on February 28, 1953. A copy can be found in the library of the Université de Paris at Jussieu; brittle, yellowed pages with the old-fashioned blue-tinted letters of long-forgotten mimeograph machines familiar only to people who are more than half a century old today. The title page has been modified with pencil and pen: the originally typed "Alexander" changed to "Alexandre", Laurent Schwartz crossed out as President of the jury and filled in as referee, the President's position devolving to Henri Cartan. The third jury member is misnamed as Georges, rather than Gustave, Choquet. Professors Schwartz and Dieudonné are concisely thanked for the "constant stimulation" they provided. Bernard Malgrange, who recalls attending the defense with a broken leg from a skiing accident, says that Grothendieck was already thinking of changing to a different mathematical area, complaining that "The subject of topological vector spaces is dead". The topic of his minor thesis was sheaves: no original research was expected in a minor thesis, but the work he did was going to be very useful to him when finally, a year or two later, he decided exactly what he wanted to work on next.

Later, when Grothendieck's thesis was published as a Memoir of the American Mathematical Society, he added a dedication which does not appear in the original: "An meiner Mutter, Hanka Grothendieck, in Verehrung und Dankbarkeit" (to my mother Hanka Grothendieck, with veneration and gratefulness).

The Grothendieck style

When mathematicians evoke Grothendieck's work, they usually think of algebraic geometry. His massive output in the subject has largely overshadowed his early work in functional analysis, which is considered as something in the nature of a youthful "trip abroad", preceding the adult stage of settling down to a serious job.

Yet it is interesting to seek for those signs and traces of the famous Grothendieck style in his early work. If Grothendieck's research approach can be characterized in a few broad strokes, these would be: maximum generality, exhaustive exploration, great abstraction, and development of foundational theories. Calculations are absent from his work (as someone pointed out, the only numbers that appear are those used to number successive paragraphs), and examples and solved problems are few; these arise essentially only as starting points for a train of thought which can end up explaining or solving them by placing them in their natural position within a very broad framework.

One of the defining traits of Grothendieck's work is the deep search for patterns and symmetries, typically expressed by the study of "functorial" properties*. Another -

* Serre has suggested that this is one reason for which, although Grothendieck's methods gave spectacular results in algebraic geometry, they would probably not have been as fruitful in number theory, a domain in which unexpected (pretty or pathological according

perhaps of all features of his work the most distinctive - is that of replacing the study of objects (spaces and their points, for example) by the study of the possible morphisms between these objects, and trying to read off the "shape" or "nature" of the objects from the information given by the maps.

All of these traits are strongly present in the work that Grothendieck did in Nancy. In a lecture at the IHES, P. Cartier described the genesis of nuclear spaces from a different angle than Schwartz's in the passage quoted above. While Schwartz mentions the two topologies that could be placed on $E \otimes F$, Cartier describes Grothendieck's definition of two different tensor products, the $\overset{\wedge}{\otimes}$ product** such that $(E \overset{\wedge}{\otimes} F)' = \mathcal{L}(E, F')$ (where F' denotes the dual space of F, according to the notation used by Schwartz and Grothendieck), and the $\overset{\vee}{\otimes}$ product defined by Grothendieck, such that $E \overset{\vee}{\otimes} F = (E' \overset{\wedge}{\otimes} F')'$. The old tensor product is right-exact, the new one left-exact; using both tensor products, one can express the definition of nuclear spaces as being those such that the two tensor products are equal, i.e. $E \otimes F$ is both right- and left-exact; a statement that is essentially functorial in nature. Cartier indicated that this work, in the course of which Grothendieck made full use of the nature of a Fréchet space as a projective limit and Dieudonné's observation that $\mathcal{D}$ is an inductive limit, together with work that he did shortly afterwards on Banach's approximation problem, in the course of which, without solving it (it turns out that there exist Banach spaces without the approximation property), he wrote down some thirty or forty equivalent properties, led him naturally towards the whole idea of abstract formalism of inductive and projective limits on which he concentrated when he later left functional analysis to devote himself to homological algebra.

A typical example of Grothendieck's focus on morphisms rather than spaces – possibly the very first one in all of his work – can be found in a result that he published in the Canadian Mathematical Journal*. A classical theorem of Dunford and Pettis showed that any L-space X has the property that every weakly compact linear operator from X to a Banach space is completely continuous. The main theorem in Grothendieck's Canadian paper is a generalization of this result to all $C(K)$ spaces. But it is his approach to the problem which differs from all past work; in a very simple sense, he stood the problem on its head. He started by defining a class of Banach spaces *characterized by properties of operators acting on them*; namely, he took the property that Dunford and Pettis had proven for L-spaces, dubbed it the "Dunford-Pettis property", and considered the class of all Banach spaces having that property. Thus, instead of starting with a particular type of Banach space having some known properties, and trying to prove that those properties must imply the Dunford-Pettis property, Grothendieck tried to figure out what properties of Banach spaces were *implied* by the assumption that they did possess the Dunford-Pettis property. He found several, from which he was able to conclude his main result about

---

to one's point of view) phenomena turn up all the time. Grothendieck was not attracted by this aspect of mathematics, which Serre cherished.

** This tensor product was studied shortly before Grothendieck began work on the subject by Schatten, a student of von Neumann.

* Sur les applications linéaires faiblement compactes d'espaces du type $C(K)$, *Canad. J. Math* **5** (1953), 129-173.

$C(K)$ spaces in particular.**

In his thesis report, Schwartz mentioned this and other results by Grothendieck published separately from his thesis, in the following terms.

Il faut enfin signaler que ce travail considérable n'est pas comme il est habituel pour la thèse, le premier travail de l'auteur. Grothendieck a déjà à son actif plusieurs mémoires importants, dont chacun pourrait constituer une thèse: étude des espaces de fonctions holomorphes, des applications linéaires faiblement compactes, des espaces du type DF, sans compter des articles plus courts mais non moins intéressants. Toutes ces publications sont relatives à la théorie des Espaces vectoriels topologiques, où l'auteur a acquis une telle virtuosité qu'il n'est pas exagéré de le considérer comme le premier spécialiste mondial en la matière.[8]

Finally, there is the telling (and extremely amusing, for which reason we provide a bit more than what strictly concerns Grothendieck) "Tribu" newsletter from the Bourbaki meeting of March 1952, at which Grothendieck was once again present, this time as the only cobaye.

Sur ses vieux jours Notre Maître se sent l'âme champêtre, fuit le bruit et la poussière des villes, aspire à être assis à l'ombre des forêts ou au soleil des glaciers. Rien ne le réjouit tant que d'entendre Dieudonné parler de devenir gentleman-farmer, et de "faire valoir". Le calme des Vosges n'empêcha cependant pas les altercations; il est vrai que l'étymologie de Bourbaki ne le prédispose pas à la mansuétude ("bachi"="chef", "vour"="tueur": chef des tueurs!). Ainsi Cartain fut accusé d'être inconsciemment de mauvaise foi, et un alexandrin stigmatisa ses errements

"Qui sème le foncteur récolte la structure."

Dieudonné se demande, à l'étonnement général, "comment on peut dire des choses sensées quand on ne fait que de l'algèbre".

Mais surtout un drâme naquit de l'accouchement laborieux des EVT. Désireux de surmonter les réticences de l'opposition, le Haut Commissariat tenta une manœuvre de chantage: il fit venir Grothendieck! On espérait effarer à tel point les Congressistes qu'ils seraient prêts à avaler tonneau sur tonneau par peur de subir une rédaction Grothendieckienne. Mais les logiciens veillaient: ils apprirent à Grothendieck que, si tous les ensembles vide sont égaux, certains du moins sont plus égaux que d'autres; le pauvre en devint fou furieux, et rentre à Nancy par le premier train.[9]

Few cobayes receive mention in "La Tribu" – let alone this kind of mention! There is no doubt about it; barely three years after the start of his studies, he was not only the world's premier specialist in his chosen domain, but he was already the mathematician he was to become: Alexandre Grothendieck.

** The source of these remarks and much more fascinating detail is the article by J. Diestel in [ref]; Diestel calls this approach, defining classes of Banach spaces by properties of the operators acting on them "a first".

An end and a beginning: Fall 1952

By the time that Grothendieck had discovered the bulk of his results concerning nuclear spaces, Schwartz and Dieudonné were convinced that, as a future mathematical star, he needed a permanent position which would provide him with the best possible working conditions as soon as possible.

A position in France was out of the question; the position "Attaché de Recherches" at the CNRS and the modest stipend it provided him was intended only as a sort of temporary solution, a scholarship to support him through his doctoral degree, and unfortunately, being stateless, he would have no access to a university position in France. The CNRS had some flexibility, and they hoped to be able to negotiate something for him eventually, but Grothendieck needed a job at once. Also, he needed to get away from Nancy.

As Godement explains it, "although Grothendieck was perfectly content there, he realized very well that the intellectual level of the population wasn't brilliant, and even the mathematics department was hardly Harvard or Princeton. Not only that, but Paris wasn't any better than Nancy at that time. The only person really doing mathematics was Henri Cartan." In saying this, Godement wasn't referring to the constellation of rising stars which was very soon going to effect a fantastic and total reformation of French mathematics; he was speaking of the older generation, those who could have been guides and mentors for someone of Grothendieck's capabilities. Brilliant as they were, Grothendieck had already outgrown Dieudonné and Schwartz; he had mastered their work to the last comma, and they, instead, were the ones poring over his new results. "Schwartz's Fields Medal didn't impress him in the least. We all knew that Schwartz was a good mathematician, we couldn't care less about his medal," says Godement, who was not unlike Grothendieck in his complete disregard for social rank. "Anyway, even if Grothendieck had met Hilbert, he'd just have said 'How are you?' He didn't have any feeling of inferiority, even compared to the greatest mathematicians ever. It was obvious that he already considered himself as a very great mathematician. He was very sure of himself."

The astounding mathematical growth he had undergone in Nancy, and the absolute confidence that it gave him, is reflected in the snatches of his writing that can be found from 1952. While his first letters to Dixmier cited above were formal and respectful, a letter from May 1952 starts abruptly: "Cher Dixmier, Je n'ai jamais prouvé ni prétendu avoir prouvé le théorème dont tu parles, et qui d'ailleurs est faux."[10] His impatience, his annoyance at being misunderstood, and his complete lack of concern about appearing rude, transpire clearly. A new side of his character was starting to show.

From the early fall, while Grothendieck was engaged in writing up the final version of his thesis, Dieudonné and Schwartz were occupied in trying actively to obtain a position for him in Brazil, a place where Schwartz had many close mathematical relations. It seems that they first made an effort to find a position for him at the University of Rio de Janeiro, where Paulo Ribenboim had returned already in July. A letter dated September 30, 1952 and signed by Dieudonné, Schwartz and C. Ehresmann, addressed to "Monsieur le Recteur de l'Universidade do Brasil" (now UFRJ) recommends the creation of a second chair in Analysis*. A letter from Dieudonné (now in Ann Arbor) to Grothendieck in October 1952

* This information and much more was communicated to me by Alberto Azevedo, who

discusses the possibility of his spending the coming academic year in Princeton (chiefly warning him of the serious difficulties he would be likely to face in trying to obtain a visa). A letter from November discusses the possibility of publishing Grothendieck's thesis as a *Memoir* of the AMS, and commiserates with an apparent complaint about the inactivity of the mathematical life in Nancy while Dieudonné, now on his way to Chicago, was roaming about the Americas.

As a letter from January 1953 confirms, it was the University of São Paulo, where André Weil (unwelcome in France after having avoided military service) had spent two years in 1945-1947, and where Schwartz and Dieudonné also visited regularly, that ended up offering Grothendieck him a two-year position for the academic years 1953 and 1954*. Dieudonné advises him to submit an article to the journal *Summa Brasiliensis*, saying "Si vous partez au Brésil, il ne sera pas mauvais que vous ayiez un travail à l'impression là-bas",[11] and goes on to give him a clearly worded warning about difficulties that his forthright nature might encounter in a foreign culture: "Si j'ai cru devoir vous mettre en garde en ce qui concerne votre comportement vis-à-vis des Brésiliens, c'est que je connais tout de même votre caractère entier et vos réactions à certaines actions ou certaines gens; je ne trouve pas du tout ça antipathique, mais il faut que vous sachiez que là-bas la mentalité est telle que lorsqu'on ne loue pas quelqu'un de façon hyperbolique, c'est presque considéré comme une insulte!!"[12] And, quoting Grothendieck as having accused him of being "seduced by American gold", Dieudonné ends tranquilly by observing that yes, he is no hero, but Americans are serious, honest and hard-working and he wouldn't mind finding himself at the same American university as Grothendieck some day.

By early 1953, then, Grothendieck was readying himself to leave Nancy for good - though not without leaving a trace of himself behind. The Japanese mathematician Reiji Takahashi**, one of the first redactors of the Cartan seminar, arrived in Nancy very shortly after Grothendieck had left, and found that his spirit was still present everywhere. At a dance during a party at the home of Delsarte, the father of two charming daughters, Takahashi noticed his host engaged in a heated discussion with a handsome young man in a corner, ignoring both the girls and the assembled guests. Wondering what could be getting them so excited, Takahashi joined them, only to hear Delsarte emitting sharp criticism of the mathematical style of the mysterious Grothendieck, with a young Jean-Pierre Serre ardently defending it. Weeks later, Takahashi entered a post office to mail a letter, and the employee behind the window happened to notice that he was a mathematician. "So you know Mr. Grothendieck?" enquired the employee with interest. Takahashi admitted that he didn't. "He was very strange," he heard the employee murmur, before returning to his other tasks.

---

tried to discover all the details of Grothendieck's visit to São Paulo, published an article in Portuguese on the subject [ref] and patiently tried to answer the many questions I addressed to him.

* The academic year in Brazil runs from March to December, with January and February being their summer vacation.

** Hiroaki Nakamura was kind enough to contact Reiji Takahashi at my request, and have a talk with him about Grothendieck, during which he noted down the amusing anecdotes related here.

What Grothendieck left behind him in Nancy went beyond mathematics and mathematicians. At some point during his stay there, on the last of his many searches for a room to rent, he met Aline Driquert, moved into her house, and succumbed to what, much later, he would call her "sex appeal".

Aline Driquert (also called Marcelle), was an extremely unusual woman, of a type calculated to appeal to Grothendieck's sensitivities. For one, she was a good fourteen years older than he was; not his mother's age, certainly, but still an attractive "older woman" in his eyes. For another, she was a real earth-mother, with a brood of four children ranging in ages from 2 to 17 running around her house. And for a third, she was as much of a social rebel as anyone that Grothendieck had ever met - an anarchist in lifestyle, if not politically, as Grothendieck's father Sascha had once described his mother Hanka, to her pride. Aline worked as a shorthand-typist in a military tribunal, and rented out a room in her house to make ends meet. Like Grothendieck himself, all of her children bore their mother's last name; none had an identified father, nor did it even appear that any of them had the same father. Husbands were not part of Aline's life as they had not been part of Hanka's; her "I can do what I want" attitude was an echo to Hanka's oft-repeated "Ich kann machen, was ich will". Hanka, apparently, did not think much of Aline Driquert, which may not be very surprising - but after hearing Grothendieck enthuse about her, the perspicacious Marie-Hélène Schwartz told her young daughter: "He's going to have a child with her".

So during that last winter in Nancy, while Grothendieck was busy putting finishing touches to thesis, organizing the defence, submitting articles, and above all preparing himself and his mother for the upcoming definitive move to Brazil, he lived at Aline's house; and in January 1953, she fell pregnant. It wasn't a very good moment, just on the eve of his departure, but Aline was not the type to calculate her own advantage. She later told Serge, the son she bore to Grothendieck, that of all the men she had loved in her life, Grothendieck was the only one who really counted; after him, there was never anyone else. Serge was also given his father's last name, which indicates that Grothendieck must have officially recognized the child as his (a formality performed in the town hall). Aline asked nothing from Grothendieck, and did nothing to try to prevent him from leaving. She would have been aware that he had his mother to support and no source of income in France; in any case he had no choice but to go. And she was used to raising children and earning her living by herself.

When she said goodbye to Grothendieck, she was still in love with the young man of whom she had seen only the very best side. She expected him to be interested in the child he had fathered, if only from a distance, and to lend her a helping hand when he would be able to. She had no idea, and in fact probably Grothendieck himself had no idea, of the rabidly angry, bitter and heartless aspect of his character that would come to light only in the months and years after the baby's birth.

Translation of the French quotations

[1] He first gave Dieudonné an article of fifty or so pages, on "Integration with values in a topological group". It was correct, but absolutely uninteresting. Dieudonné, with the (always temporary) aggressiveness he was capable of, gave him a memorable scolding, claiming that one shouldn't work that way, generalizing just for the pleasure of generalizing. The problem one considered had to be difficult, and applicable to the rest of mathematics (or other sciences); his results would never be useful to anyone for anything.

[2] If ever I saw a mathematician make use of a powerful and elemental "power to encourage", it is certainly him! I've never thought about it again until this moment, but I remember that that was the way in which he had already welcomed my very first results in Nancy, solving some questions that he had asked with Schwartz (on the spaces $\mathcal{F}$ and $\mathcal{LF}$). They were very modest result, nothing of genius, nothing extraordinary, to be sure; one might say there was nothing in them to cause such marveling.

[3] "I can understand that you are a little disappointed to see that so few questions actually admit a positive answer, but your thesis will nonetheless be a very precious piece of work, which at least does provide *an* answer to a number of questions that were natural to ask, and which required a lot of ingeniousness to solve; besides, you have a number of theorems which, in my opinion, are the only deep results in the theory since Banach, and you need have no worries about the value of your thesis: but I do agree with you that after this, you will have definitively killed the subject."

[4] "I thought I had seen that the $\phi_{x,y}$ are indeed almost periodic flows, but I can't see why any more now that I'm writing to you, so that it seems very possible to me that I was wrong – though I'm not sure."

[5] I looked at the other results of my previous letter and this one with enough care to be absolutely certain!

[6] At the end of July in Brazil, I received a very disappointed letter from him: there were two locally convex topologies on $E \otimes F$, both equally natural, but different! So there was nothing interesting to say. I didn't know what to answer him. It was clear that on $\mathcal{D}' \otimes F$ there was only one obvious topology. But difficulty and defeat can often be a source of victory. Two weeks later, I received a new and triumphant letter: the two topologies coincided in the case of $\mathcal{D}' \otimes F$. There are certain locally convex spaces, which he called "nuclear", such that for every F, the two topologies on $E \otimes F$ coincide. Everything became clear.

[7] The work presented here is very long, but absolutely not "diluted". The theorems stated are difficult, and require a great deal of ingeniosity. Many original ideas, a perfect technique (every proof is as short as possible, using exactly the adequate methods), a very clear and ordered conception (the first draft was very close to being the final draft), all these are excellent qualities. Certainly, there are also some defects. The statements of the theorems are too long, so too heavy; it would have been better to shorten them,

putting parts of them as remarks or in the proofs. Also, he lacks the discernment which makes it possible to separate, among results which are new and hard to prove, those which are essential or only secondary for the future of mathematics, which results in a massive presentation extremely difficult to read for non-specialists. These defects of "youth" are pedagogical in nature and should not be neglected, but they remove nothing of the character and exceptional value of the thesis. Finally, even if the author has until now confined himself to a single branch of mathematics, it must be said that the harvest of results obtained there have justified this specialization, and that Mr. Grothendieck does possess a solid culture, and abilities which will surely allow him to find results in other domains as well.

[8] I must point out, finally, that this considerable piece of work is not, as is usual for a thesis, the author's first piece of work. Grothendieck already has several important articles of which each could constitute a thesis: a study of spaces of holomorphic functions, weakly compact linear maps, spaces of type $\mathcal{DF}$, without even counting shorter but equally interesting articles. All these publications are related to the theory of topological vector spaces, in which the author has acquired such a level of virtuosity that it is no exaggeration to consider that he is now the greatest specialist in the world.

[9] As he ages, our Master feels his soul becoming countrified, flees the noise and dust of cities, and aspires to be seated in the shade of forests or in the sunshine of glaciers. Nothing makes him happier than to hear Dieudonné speak of becoming a gentleman-farmer and "make value". The calm of the Vosges did not, however, prevent quarrels: it is true that the etymology of Bourbaki does not incline him to indulgence ("bachi"=chief, "vour"= killer: chief of killers!) Thus, Cartan was accused of being unconsciously insincere, and an Alexandrine verse stigmatises his errings: "He who sows the functor reaps the structure."

Dieudonné asks, to general surprise, "how one can say anything sensible if one does only algebra".

But above all, a drama arose from the laborious birth pangs of Topological Vector Spaces. Desirous of overcoming the reticence of the opposition, the High Commissary attempted a blackmail maneuver: it called upon Grothendieck! The hope was to cause such a panic in the Congressists that they would be ready to swallow anything for fear of having to undergo a Grothendieckian write-up. But the logicians were standing guard: they taught Grothendieck that even if all empty sets are equal, some, at least, are more equal than others; the poor fellow went raving mad and took the first train back to Nancy.

[10] Dear Dixmier, I never proved or claimed to have proved the theorem you mention, which in any case is wrong.

[11] If you leave for Brazil, it wouldn't be a bad thing to have an article already being printed over there.

[12] If I think it's necessary to warn you about your behavior towards Brazilians, it's because I know your absolute character and your reactions to certain actions and certain people: I don't find that at all unlikable, but you must know that over there, the mentality is such that if you don't praise someone to the skies, it's practically an insult!!

[13] It was in 1952, I believe, when Serre came to Nancy (where I remained until 1953) that he started to become a privileged interlocutor for me.

Attached Documents

Photocopy of Grothendieck's very first Note aux CRAS.
Photocopy of first pages of Grothendieck's thesis.
Schwartz's report on Grothendieck's thesis
Grothendieck's letters to Dixmier
Pictures from Nancy (addresses)
Pictures of Schwartz, Dieudonné, Delsarte, Takahashi, Ribenboim