SOLUTION HOMEWORK 3

(HW_3_1) [Sec. 3.1, #4]
In my perusal of a zip code directory, I found no 00000, nor did I find any zip codes with four zeros, a fact which was not obvious. Thus possible X values are 2, 3, 4, 5 (and not 0 or 1). X = 5 for the outcome 15213, X = 4 for the outcome 44074, and X = 3 for 94322.

(HW_3_2) [Sec. 3.2, #12]
(a) $p_2(x)$ is the acceptable probability function, because all the probabilities are between zero and one and if probabilities add up to 1, whereas $p_1(x)$ and $p_3(x)$ are not acceptable because the summation of probabilities add up to a number which is less than one and greater than one respectively.
(b) $P(2 \leq X \leq 4)= P(X=2)+P(X=3)+P(X=4)=0.5$

\[ P(X \leq 2) = 1 - P(X \geq 3) = 0.6 \]

\[ P(X \neq 0) = 1 - P(X \geq 1) = 0.6 \]

(c) $p(x) = c (5 - x)$, then $\sum p(x)=1$. Hence, $\sum c (5 - x) = 1$. This implies that

\[ c((5-0)+(5-1)+(5-2)+(5-3)+(5-4))=1, \text{that is } c(15)=1 \text{ and so } c=1/15. \]

(HW_3_3) [Sec. 3.2, #13]

a. $P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = .10+.15+.20+.25 = .70$

b. $P(X < 3) = P(X \leq 2) = p(0) + p(1) + p(2) = .45$
c. \( P(3 \leq X) = p(3) + p(4) + p(5) + p(6) = .55 \)
d. \( P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = .71 \)
e. The number of lines not in use is \( 6 - X \), so \( 6 - X = 2 \) is equivalent to \( X = 4 \), \( 6 - X = 3 \) to \( X = 3 \), and \( 6 - X = 4 \) to \( X = 2 \). Thus we desire \( P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = .65 \)
f. \( 6 - X \geq 4 \) if \( 6 - 4 \geq X \), i.e. \( 2 \geq X \), or \( X \leq 2 \), and \( P(X \leq 2) = .10 + .15 + .20 = .45 \)

- **(HW 3 4) [Sec. 3.3, #28]**

a. \( E(X) = \sum_{x=0}^{4} x \cdot p(x) = (0)(.08) + (1)(.15) + (2)(.45) + (3)(.27) + (4)(.05) = 2.06 \)
b. \( V(X) = \sum_{x=0}^{4} (x - 2.06)^2 \cdot p(x) = (0 - 2.06)^2(.08) + \ldots + (4 - 2.06)^2(.05) \\
= .339488 + .168540 + .001620 + .238572 + .188180 = .9364 \)
c. \( \sigma_x = \sqrt{.9364} = .9677 \)
d. \( V(X) = \left[ \sum_{x=0}^{4} x^2 \cdot p(x) \right] - (2.06)^2 = 5.1800 - 4.2436 = .9364 \)

- **(HW 3 5) [Sec. 3.3, #31]**

a. \( E(X) = (13.5)(.2) + (15.9)(.5) + (19.1)(.3) = 16.38, \) \( E(X^2) = (13.5)^2(.2) + (15.9)^2(.5) + (19.1)^2(.3) = 272.298, \) \( V(X) = 272.298 - (16.38)^2 = 3.9936 \)
b. \( E(25X - 8.5) = 25 E(X) - 8.5 = (25)(16.38) - 8.5 = 401 \)
c. \( V(25X - 8.5) = V(25X) = (25)^2V(X) = (625)(3.9936) = 2496 \)
d. \( E[h(X)] = E[X - .01X^2] = E(X) - .01E(X^2) = 16.38 - 2.72 = 13.66 \)