SOLUTION HOMEWORK 5

Due Date: Wednesday, June 09, 2004

http://www.stat.ucla.edu/~dinov/courses_students.dir/04/Spr/Stat110A.dir/HWs.dir/HW5.html

(There is a total of 100 points for this assignment.)

Problem 1 (Total: 16 points; 4 points each)

a) Yes.
b) \{X \geq t\} = A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5
c) P(X \geq t) = P(A_1)P(A_2)P(A_3)P(A_4)P(A_5) = (e^{-0.05t})^5 = e^{-0.05t}
   F(t) = P(X \leq t) = 1 - e^{-0.05t}
d) f(t) = 0.05 e^{-0.05t}, X has an exponential distribution with \lambda = 0.05.

Problem 2 (Total: 16 points; 4 points each)
X~ normal distribution; P = 0.1; n=200; np = 20; npq = 18

a) P(X \leq 30) = \Phi \left( \frac{30 + 0.5 - 20}{\sqrt{18}} \right) = 0.9932
b) P(X < 30) = P(X \leq 29) = \Phi(2.24) = 0.9875
c) P(15 \leq X \leq 25) = P(X \leq 25) - P(X \leq 14) = \Phi(1.30) - \Phi(-1.30) = 0.8064
d) X = 20 - 0.44(\sqrt{18}) = 18.1332

Problem 3 (Total: 15 points; 5 points each)

a) \frac{f(y|x)}{f_x(x)} = \frac{\frac{9}{26} y^2 (x+1)^2}{\int_0^1 \frac{9}{26} y^2 (x+1)^2 dy} = 3y^2
b) \( P(Y < \frac{1}{2} | X < \frac{1}{2}) = \frac{P(0 < x < 1/2; 0 < y < 1/2)}{P(0 < x < 1/2)} = \frac{\int_0^{1/2} \int_0^{1/2} f(x, y) dy dx}{\int_0^{1/2} f_x(x) dx} = \frac{1}{8} \)

c) \( E(Y | X = x) = \int_0^1 y f(y | x) dy = \int_0^1 y (3y^2) dy = \frac{3}{4} \)

**Problem 4 (Total: 16 points; 4 points each)**
Refer to your lecture notes.

**Problem 5 (Total: 14 points; 7 points each)**

a) \( E(X) = \int_0^1 x (\theta + 1) x^\theta dx = \frac{\theta + 1}{\theta + 2} = 1 - \frac{1}{\theta + 2} \) So \( \overline{X} = 1 - \frac{1}{\theta + 2} \) \( \Rightarrow \hat{\theta} = \frac{1}{1 - \overline{X}} - 2 \)

Since \( \overline{X} = 0.80 \), \( \hat{\theta} = 3. \)

b) \( f(x_1, \ldots, x_n; \theta) = (\theta + 1)^n (x_1 x_2 \cdots x_n)^\theta \) so \( \log f = n \ln(\theta + 1) + \theta \sum \ln(x_i) \)

\( \Rightarrow \hat{\theta} = -\frac{n}{\sum \ln(x_i)} - 1 \) so \( \hat{\theta} = 3.12 \)

**Problem 6 (Total: 10 points)**

95% C.I. is \( 1.95 \pm 2.365 \left( \frac{0.22}{\sqrt{8}} \right) = (1.766, 2.134) \)

**Problem 7 (Total: 13 points)**

\( p = \frac{260}{500} = 0.52 \)

s.d. = \( \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.52)(0.48)}{500}} = 0.02234 \) (1 point)

95% C.I. is \( 0.52 \pm (1.96)(0.02234) = (0.476, 0.564) \) (6 points)

\( 0.02 = \sqrt{\frac{(0.52)(0.48)}{n}} \) so \( n = 624 \) (6 points)