HW_3_1

X = number disk drives which malfunction

(1) This is Binomial distribution with n = 11, p = 0.03.

(2) In the context of this exercise, state the assumptions required for X to have a Binomial distribution.

→ The occurrence of each disk drive’s malfunction is independent of another’s. Binomial distribution requires each trial to be independent of another.

(3) Are the Binomial assumptions satisfied here?

→ Yes. The malfunction of a disk drive should not be dependent of another disk drive.

(4) Calculate the probability that:

1. No disk drive will malfunction during the warranty period.

\[ P(X = 0) = \binom{11}{0} 0.03^0 0.97^{11} = 0.7153 \]

2. Exactly one disk drive will malfunction during the warranty period.

\[ P(X = 1) = \binom{11}{1} 0.03^1 0.97^{10} = 0.2434 \]

3. At least two disk drives will malfunction during the warranty period.

\[ P(X \geq 2) = 1 - P(X = 0 \text{ or } X = 1) = 1 - (P(X = 0) + P(X = 1)) \]
\[ = 1 - 0.7153 - 0.2434 = 0.04135 \]

4. Between 2 and 5 (inclusive) disk drives will malfunction during the warranty period.

\[ P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = \]
\[ \binom{11}{2} 0.03^2 0.97^9 + \binom{11}{3} 0.03^3 0.97^8 + \binom{11}{4} 0.03^4 0.97^7 + \binom{11}{5} 0.03^5 0.97^6 \]
\[ = 0.03763144 + 0.003491577 + 0.0002159738 + 0.00000935 = 0.04135 \]
Note: The answers of part 3 and 4 are only approximately equal. \( P(X \geq 5) \) is close to 0.

**HW 3.2**

Let \( X = \) wife's blood type, \( Y = \) husband's blood type.

The wife's blood type is independent of the husband's, so

\[
P(X = x \text{ and } Y = y) = P(X = x) \times P(Y = y)
\]

(1) What is the probability that both husband and wife have type A blood?

The wife's blood type is independent of the husband's.

\[
P(X = A \text{ and } Y = A) = P(X = A) \times P(Y = A) = .25 \times .25 = 0.0625
\]

(2) What is the probability that at least one of them has blood type AB?

\[
P(X = AB \text{ or } Y = AB) = P(X = AB) + P(Y = AB) - P(X = AB \text{ and } Y = AB)
\]

\[
= .11 + .11 - .11 \times .11 = 0.2079
\]

Or

\[
P(X = AB \text{ or } Y = AB) = P(X = AB \text{ and } Y = AB) + P(X = AB \text{ and } Y = A) + P(X = AB \text{ and } Y = B) + P(X = AB \text{ and } Y = O)
\]

\[
= .11 \times .11 + .11 \times (.25 + .15 + .49) \times 2 = 0.2079
\]

(3) What is the probability that they have the same blood type?

\[
P(X = Y)
\]

\[
= P(X = A \text{ and } Y = A) + P(X = B \text{ and } Y = B) + P(X = AB \text{ and } Y = AB) + P(X = O \text{ and } Y = O)
\]

\[
= .15 \times .15 + .25 \times .25 + .49 \times .49 + .11 \times .11 = 0.3372
\]

**HW 3.3**

<table>
<thead>
<tr>
<th>Gender \ Days of Exercise</th>
<th>A(0-1)</th>
<th>B(2-3)</th>
<th>C(4-5)</th>
<th>D(6-7)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>40</td>
<td>53</td>
<td>26</td>
<td>6</td>
<td>125</td>
</tr>
</tbody>
</table>
(1) is a female?

\[
P(\text{female}) = \frac{\text{number of female}}{\text{number of male} + \text{number of female}} = \frac{150}{275} = 0.5454
\]

(2) is a female and exercises 2-5 times a week?

\[
P(\text{female and exercises 2-5 times a week}) = \frac{\text{cell}(2, 2) + \text{cell}(2, 3)}{\text{total}} = \frac{68 + 37}{275} = 0.3818
\]

(3) is a male or exercises 4 or more times a week?

Let events \( A = \text{male} \); \( B = \text{exercises 4 or more times a week} \)

\[
P(A \text{ or } B) = P(A) + P(B) - P(\text{A and B})
\]

\[
P(A) = \frac{125}{275} = 0.4545
\]

\[
P(B) = \frac{63 + 17}{275} = 0.2909 \quad \text{(From the last row, columns C and D)}
\]

\[
P(\text{A and B}) = \frac{26 + 6}{275} = 0.116
\]

So, \( P(A \text{ or } B) = 0.629 \)

(4) is a male given that the person exercises 3 times or less a week?

Let events \( A = \text{male} \); \( B = \text{the person exercises 3 times or less a week} \)

\[
P(A | B) = \frac{P(A \text{ and } B)}{P(B)}
\]

\[
P(A \text{ and } B) = \frac{40 + 53}{275} \quad \text{(columns A and B for male)}
\]

\[
P(B) = \frac{74 + 121}{275} \quad \text{(Last row, columns A and B).}
\]

\[
P(A | B) = \frac{(40 + 53) / 275}{(74 + 121) / 275} = 0.4769
\]